UNIVERSITY OF OXFORD

Short Course on Nonlinear Acoustics

22nd International Symposium on Nonlinear Acoustics

Oleg Sapozhnikov

Part II.
Nonlinear acoustics of fluids. Perturbation modes and related phenomena. Radiation force. Specificity of nonlinear effects in the presence of shocks.

Acoustic, entropy, and vorticity modes

Equations for classical thermoviscous fluid

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right]=-\nabla p+\eta\,\Delta\mathbf{v}+\left(\varsigma+\frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{v})$$

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0$$

$$\rho T\left[\frac{\partial s}{\partial t}+(\mathbf{v}\cdot\nabla)s\right]=\kappa\,\Delta T+\frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right)^2+\varsigma(\nabla\cdot\mathbf{v})^2$$

Thermodynamic relations

$$d\rho=dp/c^2-ds\cdot\alpha_V\,T\rho/c_p$$

$$dT=dp\cdot\alpha_V\,T/(\rho c_p)+ds\cdot T/c_p$$

Fluid parameters

$$\alpha_V=-\rho^{-1}(\partial\rho/\partial T)_p$$

$$c=\sqrt{(\partial p/\partial\rho)_s}\qquad c_p=T(\partial s/\partial T)_p$$

Linearization:

$$\rho = \rho_0 + \rho', \ p = p_0 + p', \ s = s_0 + s', \ T = T_0 + T'$$

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{\eta}{\rho_0}\Delta\mathbf{v} + \frac{\varsigma + \eta/3}{\rho_0}\nabla\left(\nabla\cdot\mathbf{v}\right) - \frac{1}{\rho_0}\nabla p'$$

$$\frac{\partial p'}{\partial t} = -\rho_0 c_0^2 \nabla\cdot\mathbf{v} + \frac{c_0^2 \kappa \alpha_V^2 T_0}{\rho_0 c_p^2}\Delta p' + \frac{c_0^2 \kappa \alpha_V T_0}{c_p^2}\Delta s'$$

$$\frac{\partial s'}{\partial t} = \frac{\kappa \alpha_V}{\rho_0^2 c_p}\Delta p' + \frac{\kappa}{\rho_0 c_p}\Delta s'$$

$$\rho' = \frac{1}{c_0^2} p' - \frac{\rho_0 \alpha_V T_0}{c_p} s'$$

$$T' = \frac{\alpha_V T_0}{\rho_0 c_p} p' + \frac{T_0}{c_p} s'$$

The Helmholtz decomposition

Velocity is the sum of an irrotational (curl-free) and a solenoidal (divergence-free) vector fields:

$$\mathbf{v} = \nabla\varphi + \nabla\times\mathbf{A}$$

Vorticity mode:

$$\mathbf{v}_{vor} = \nabla\times\mathbf{A}$$

VORTICITY MODE

$$\frac{\partial \mathbf{v}_{vor}}{\partial t} = \frac{\eta}{\rho_0} \Delta \mathbf{v}_{vor}$$

$$\nabla \cdot \mathbf{v}_{vor} = 0$$

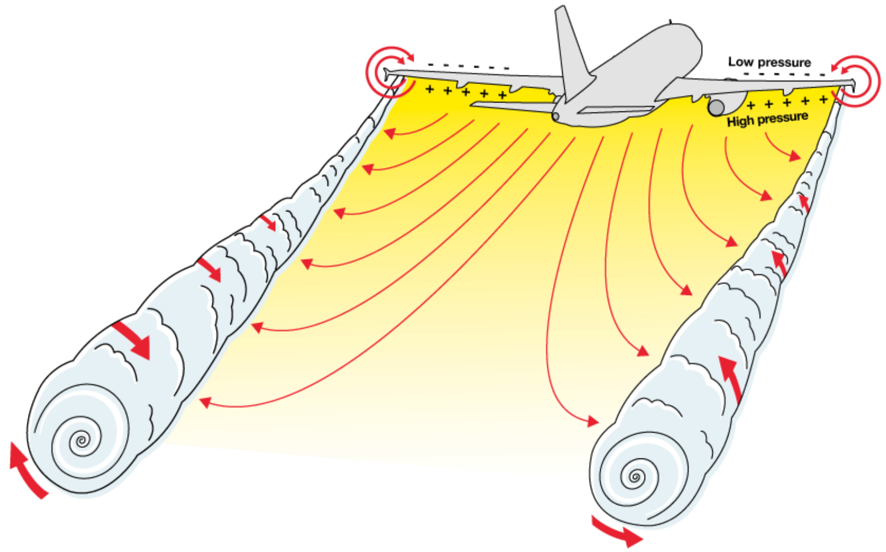
$$s_{vor} = 0$$

$$p_{vor} = 0$$

$$\rho_{vor} = 0$$

$$T_{vor} = 0$$

All aircraft generate wake vortices, also known as wake turbulence



Wake caused by a boat

POTENTIAL MOTION

$$\frac{\partial \varphi}{\partial t} = \frac{\varsigma + 4\eta/3}{\rho_0}\Delta\varphi - \frac{1}{\rho_0}p'$$

$$\frac{\partial p'}{\partial t} = -\rho_0 c_0^2 \Delta\varphi + \frac{c_0^2 \kappa \alpha_V^2 T_0}{\rho_0 c_p^2}\Delta p' + \frac{c_0^2 \kappa \alpha_V T_0}{c_p^2}\Delta s'$$

$$\frac{\partial s'}{\partial t} = \frac{\kappa \alpha_V}{\rho_0^2 c_p}\Delta p' + \frac{\kappa}{\rho_0 c_p}\Delta s'$$

$$\rho' = \frac{1}{c_0^2}p' - \frac{\rho_0 \alpha_V T_0}{c_p}s'$$

$$T' = \frac{\alpha_V T_0}{\rho_0 c_p}p' + \frac{T_0}{c_p}s'$$

ROTATIONAL MOTION

Vorticity mode

Entropy mode

Acoustic mode

ENTROPY MODE (heat diffusion)

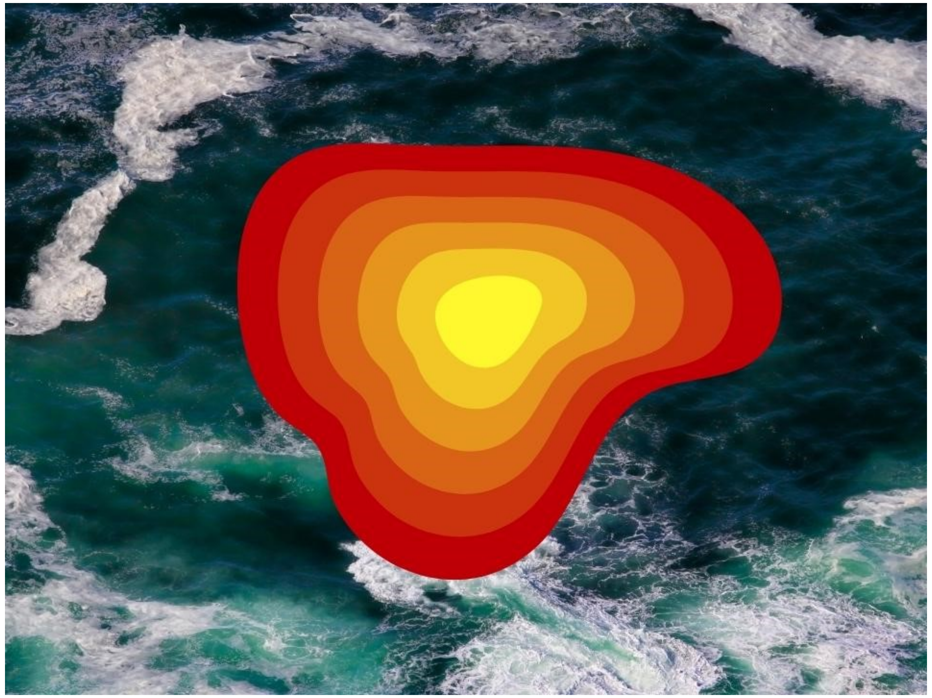
$$\frac{\partial s_{ent}}{\partial t} \approx \frac{\kappa}{\rho_0 c_p} \Delta s_{ent}$$

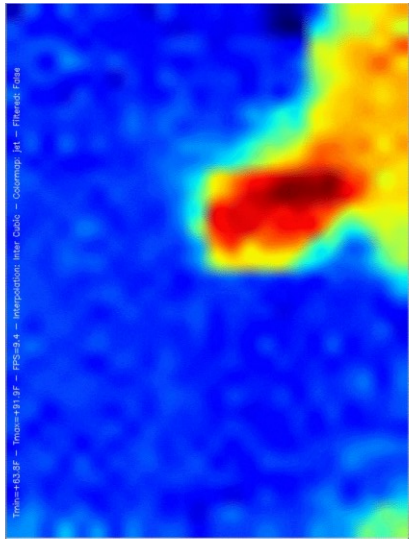
$$T_{ent} = T_0 / c_p \cdot s_{ent}$$

$$\rho_{ent} = -\alpha_V T_0 \rho_0 / c_p \cdot s_{ent} = -\rho_0 \alpha_V T_{ent}$$

$$p_{ent} \approx 0$$

$$\mathbf{v}_{ent} \approx \kappa \alpha_V T_0 / \left(\rho_0 c_p^2 \right) \cdot \nabla s_{ent}$$





ACOUSTIC MODE (sound waves)

$$\frac{\partial^2 p_{ac}}{\partial t^2} - c_0^2 \Delta p_{ac} \approx \delta \frac{\partial}{\partial t} \Delta p_{ac}$$

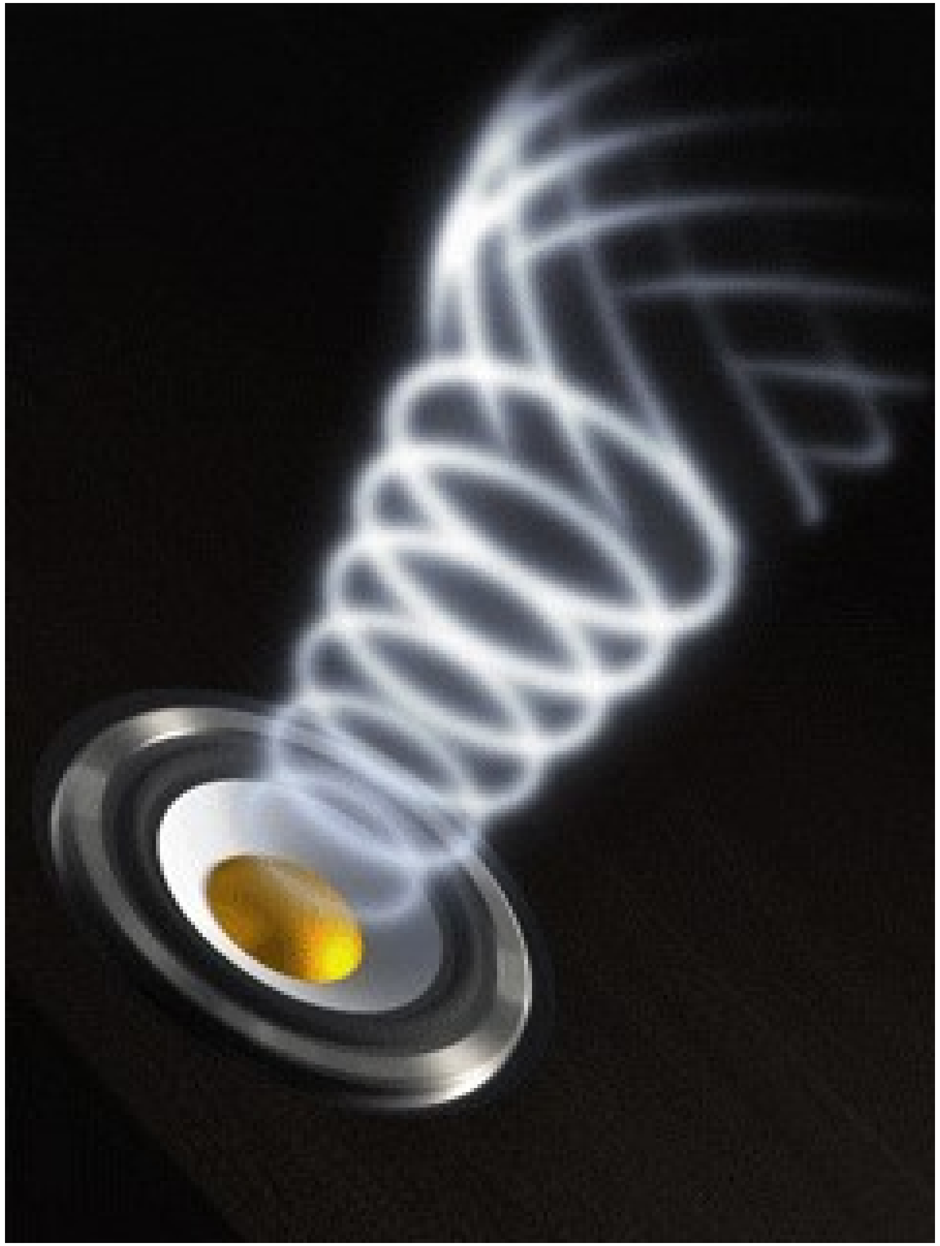
$$\frac{\partial \mathbf{v}_{ac}}{\partial t} = -\left(\frac{1}{\rho} + \frac{\varsigma + 4\eta/3}{\rho^2 c^2} \frac{\partial}{\partial t} \right) \nabla p_{ac} \approx -\frac{1}{\rho} \nabla p_{ac}$$

$$\rho_{ac} = c_0^{-2} p_{ac} + c_0^{-2} \kappa \left(c_v^{-1} - c_p^{-1} \right) \nabla \cdot \mathbf{v}_{ac}$$

$$T_{ac} = \alpha_V T_0 / \left(\rho_0 c_p \right) \cdot \left(p_{ac} - \kappa c_p^{-1} \nabla \cdot \mathbf{v}_{ac} \right)$$

$$s_{ac} \approx -\alpha_V \kappa / \left(\rho_0 c_p \right) \nabla \cdot \mathbf{v}_{ac}$$

$$\delta = \left[4\eta/3 + \varsigma + \kappa \left(c_v^{-1} - c_p^{-1} \right) \right] / \rho_0$$



Equations for classical thermoviscous fluid

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right]=-\nabla p+\eta\,\Delta\mathbf{v}+\left(\varsigma+\frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{v})$$

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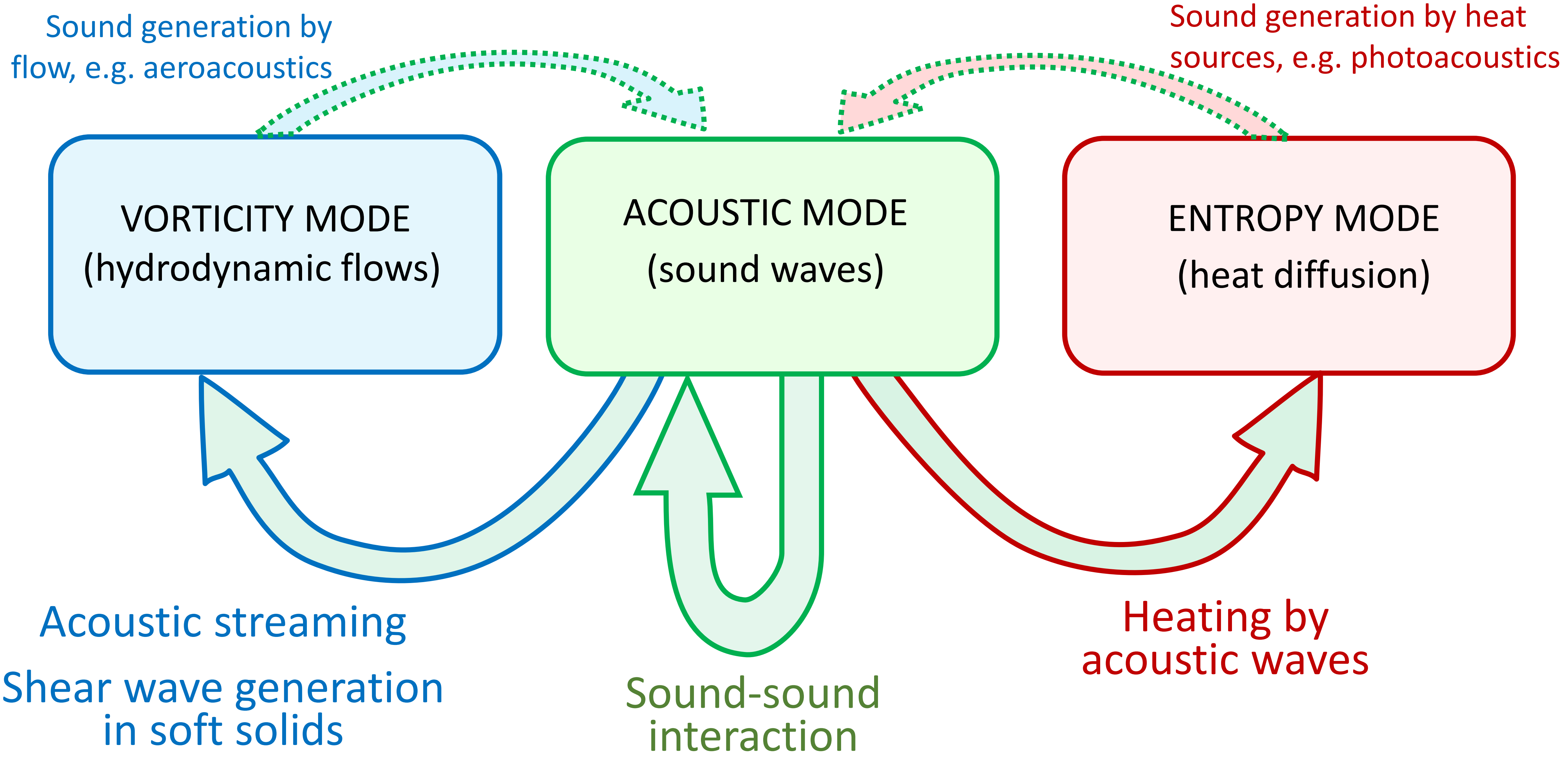
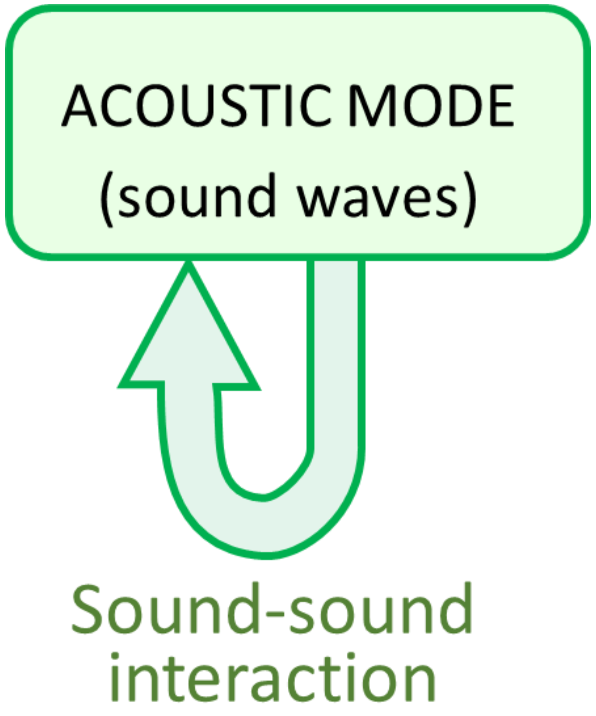
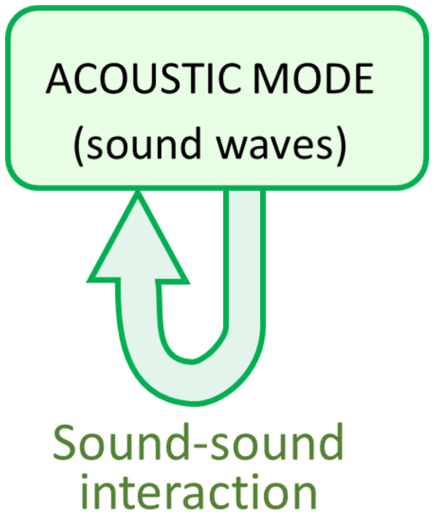
THE SUBJECTS OF STUDY IN NONLINEAR ACOUSTICS



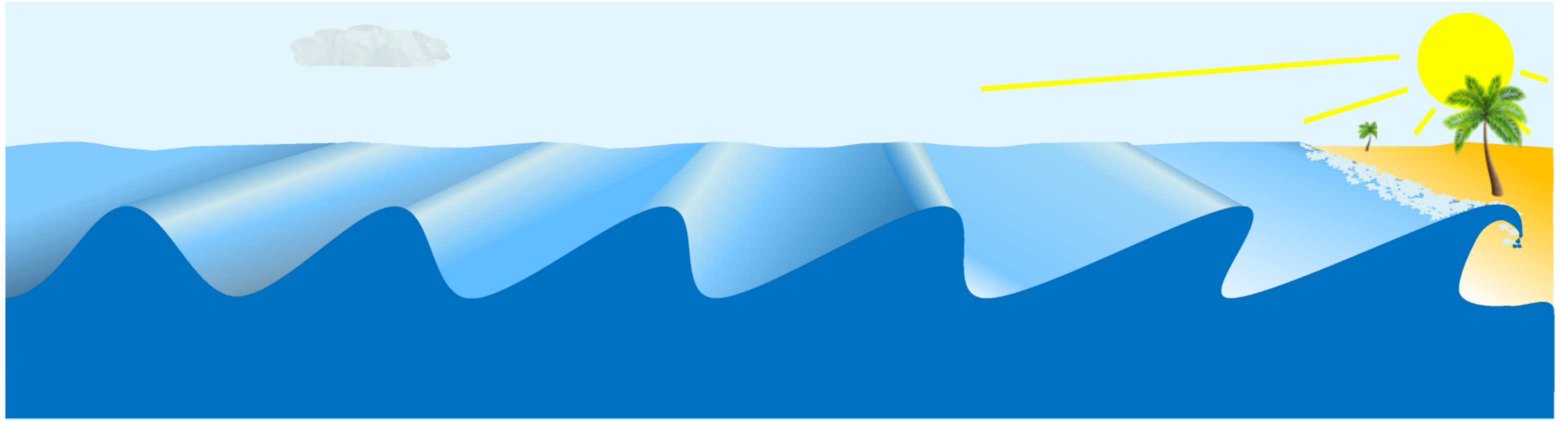
Sound-sound interaction (nonlinear phenomena within the acoustic mode)



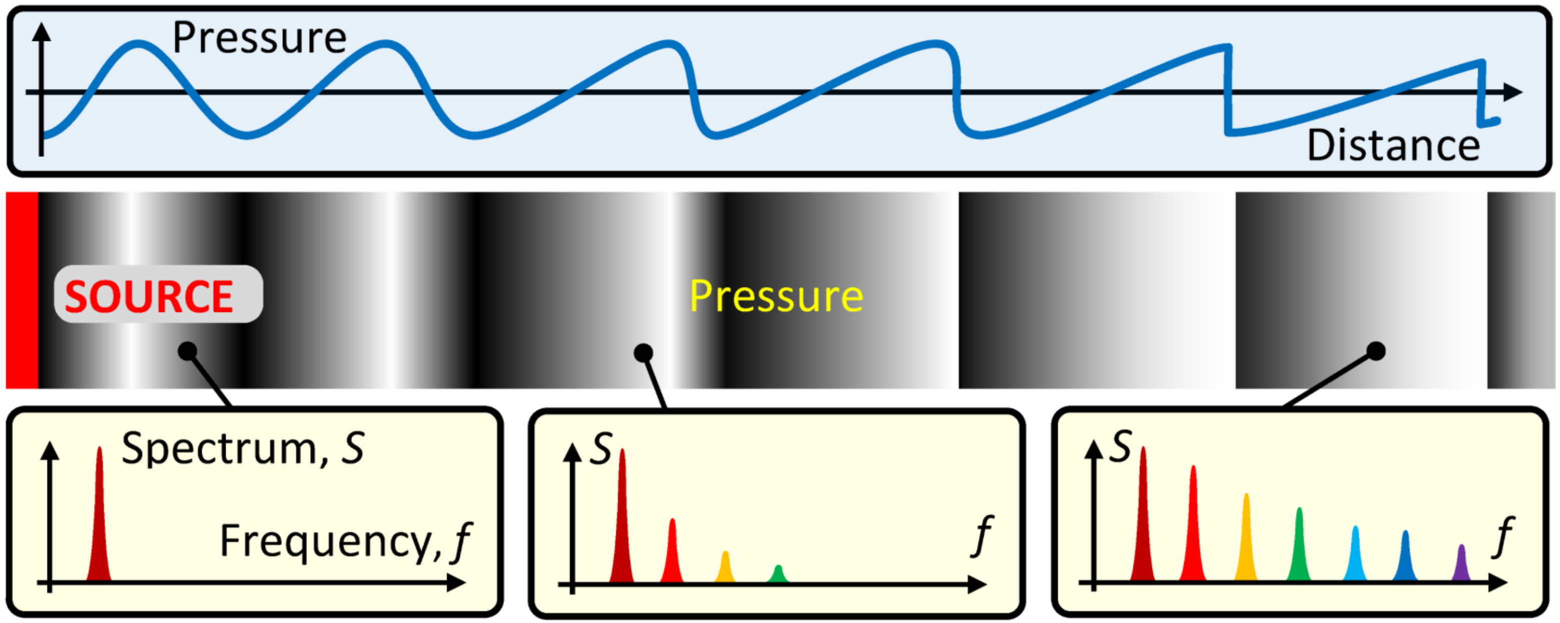
HARMONIC GENERATION



WAVE ON THE SEA SURFACE



NONLINEAR ACOUSTIC WAVE



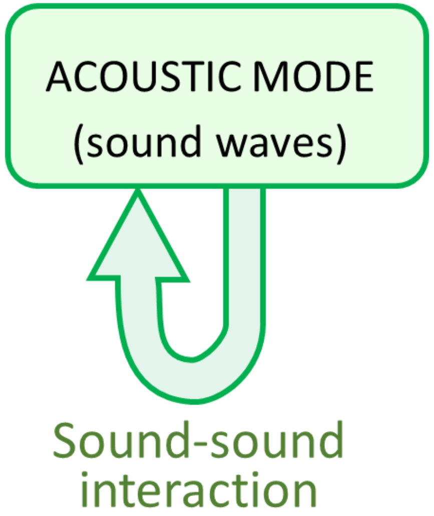
Harmonic generation as a phonon-phonon interaction

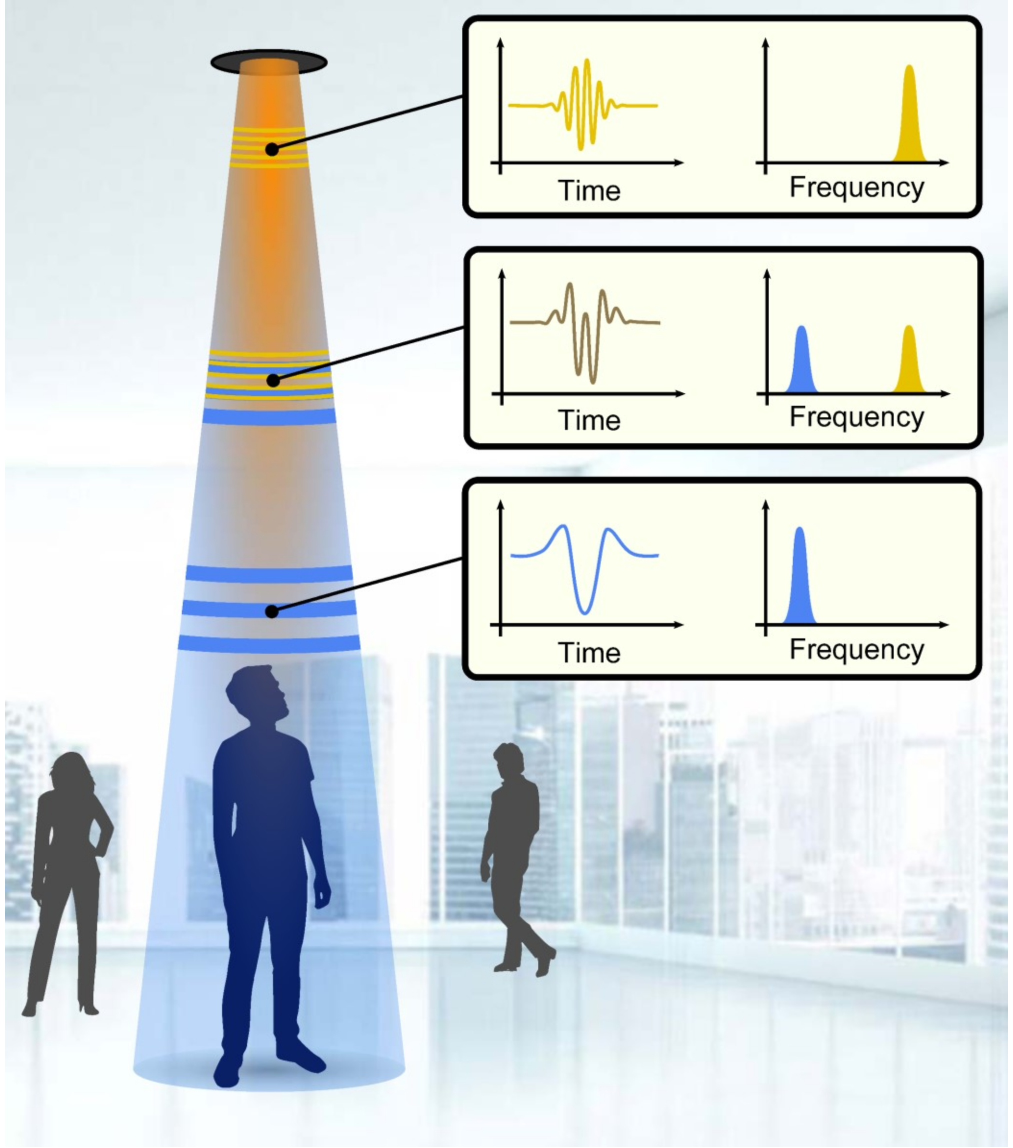
$$\hbar\omega + \hbar\omega = \hbar \times 2\omega$$

$$\hbar\omega + \hbar \times 2\omega = \hbar \times 3\omega$$

$$\omega \rightarrow 2\omega, 3\omega, \ldots$$

DIFFERENCE FREQUENCY GENERATION
Parametric arrays



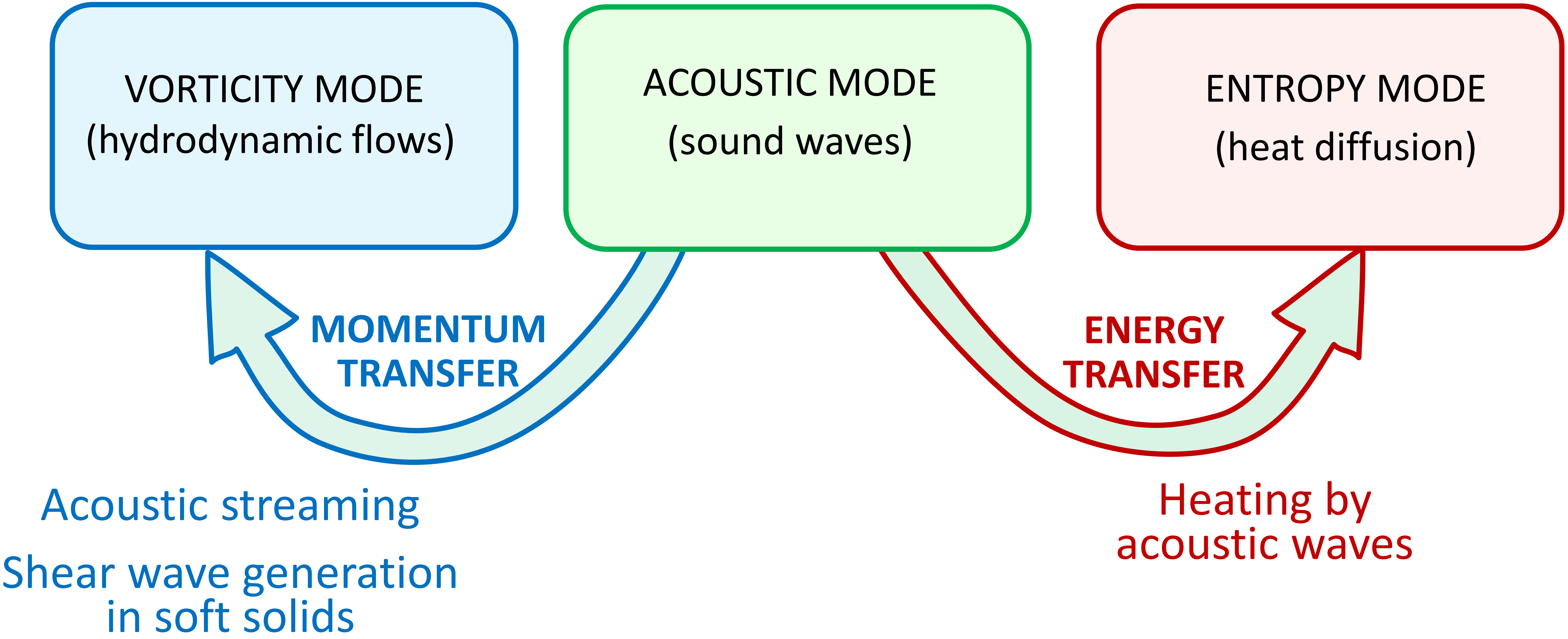


Difference frequency generation as a phonon-phonon interaction

$$\hbar\omega_2 - \hbar\omega_1 = \hbar \times (\omega_2 - \omega_1)$$

$$\omega_1 , \omega_2 \rightarrow \Omega = \omega_2 - \omega_1$$

THE SUBJECTS OF STUDY IN NONLINEAR ACOUSTICS



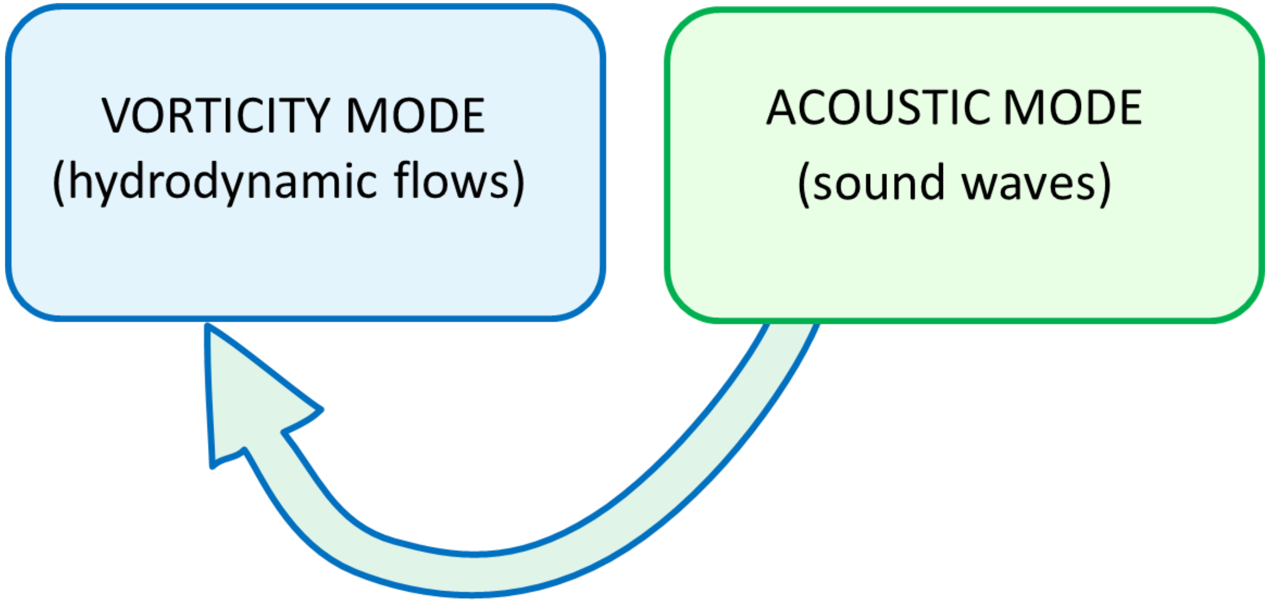
MOMENTUM VS PSEUDO-MOMENTUM

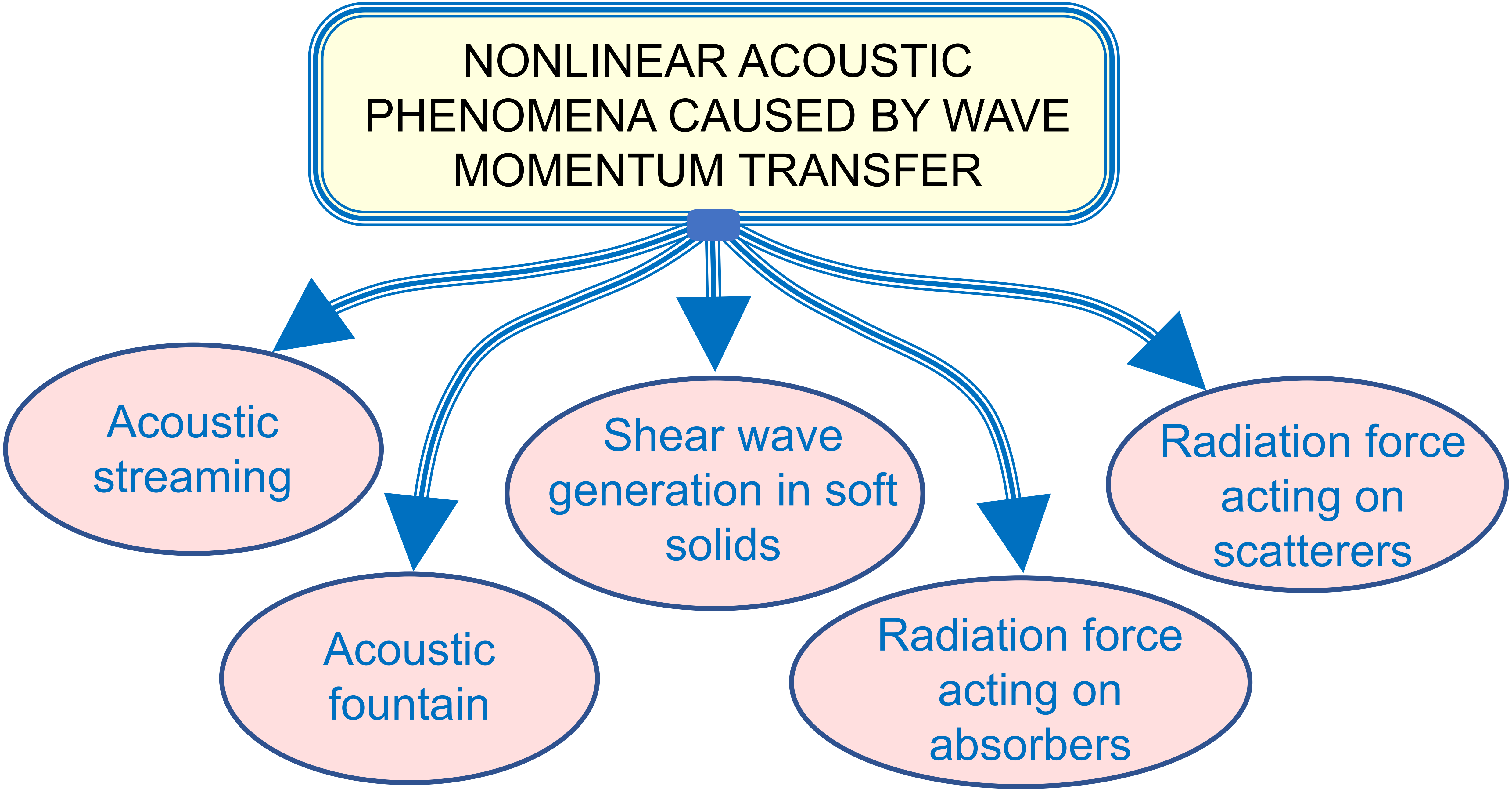
The calculation of the momentum of acoustic waves requires taking into account the nonlinear terms of the second order. However, in many practical cases (lossless medium, open space) the second-order terms cancel each other out, and it is possible to define a pseudo-momentum that is expressed by only the terms of the linear approximation.

McIntyre, J. Fluid Mech.1981 formulated ‘the pseudo-momentum rule’:
The mean forces on the obstacle are often same **as if**
(a) the waves had momentum equal to their pseudo-momentum and
(b) the medium were absent.

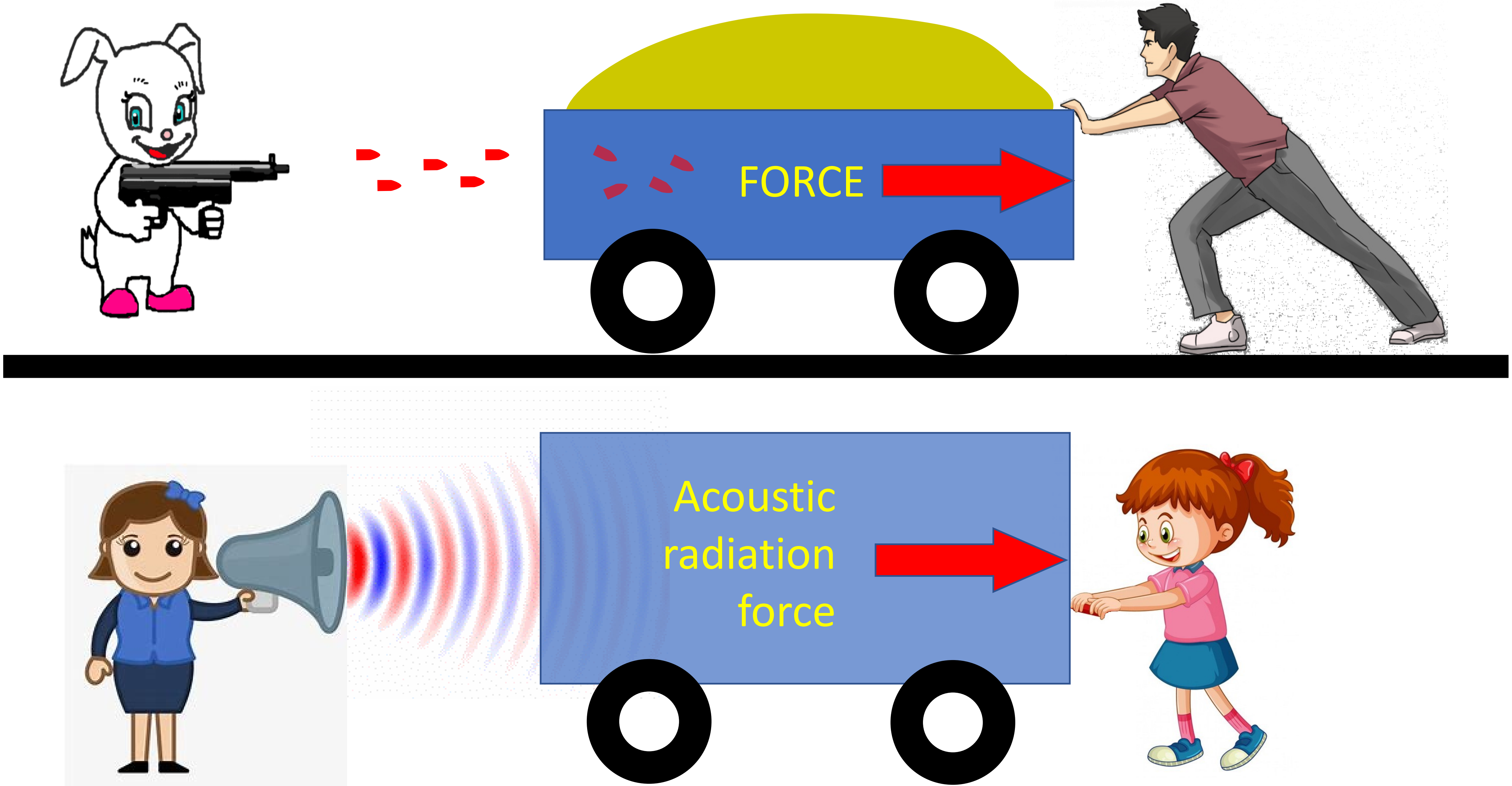
In the acoustic example of the radiation force on an absorber in a pipe with rigid walls, the pseudo-momentum rule is NOT satisfied – there is an average isotropic stress of the second order of smallness. However, if the tube walls are allowed to expand outward under the action of the radiation stress, this will lead to an adjustment of the mean pressure field, which can compensate for the isotropic component of the radiation stress, and the pseudo-momentum rule becomes valid.

Excitation of the vorticity mode by sound (transfer of momentum from sound to medium)




NONLINEAR ACOUSTIC PHENOMENA CAUSED BY WAVE MOMENTUM TRANSFER
Acoustic streaming
Acoustic fountain
Shear wave generation in soft solids
Radiation force acting on absorbers
Radiation force acting on scatterers

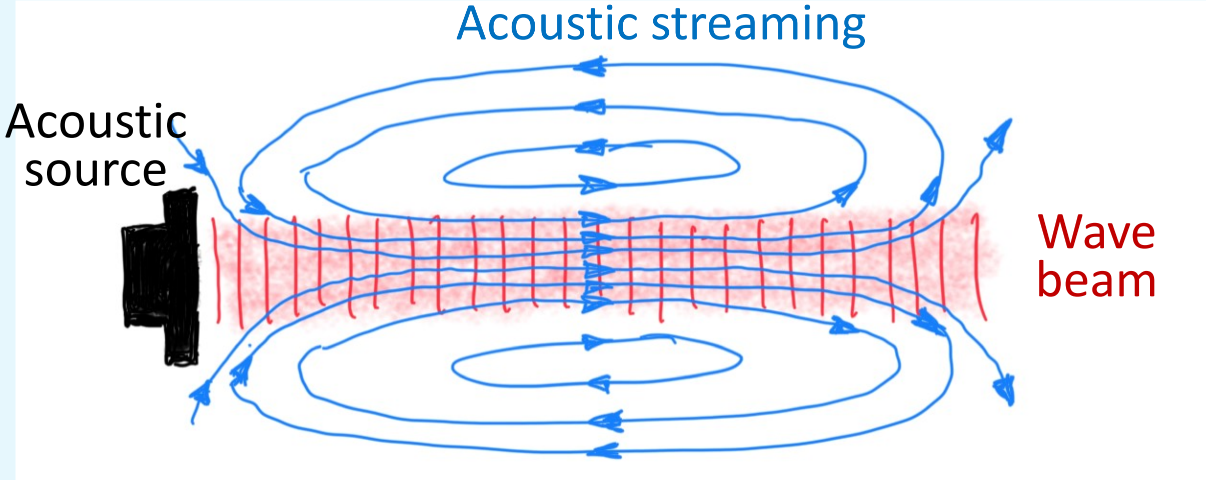
Transfer of momentum results in force



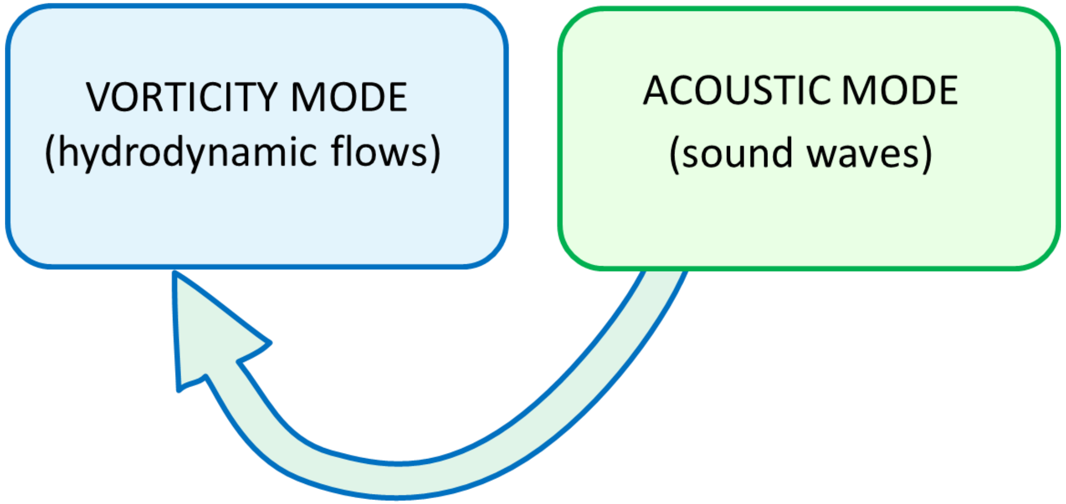
ACOUSTIC STREAMING

ECKART STREAMING

Caused by sound absorption during propagation in bulk flow



RAYLEIGH STREAMING - Caused by sound absorption near a boundary, either when sound reaches a boundary, or when a boundary is vibrating in a still medium.



ACOUSTIC STREAMING

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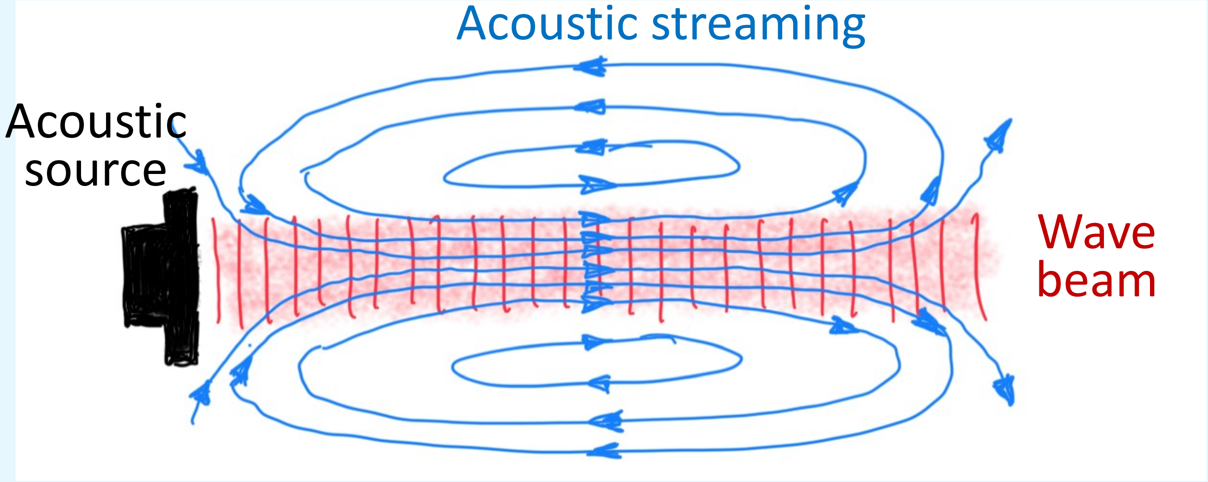
Caused by sound absorption during propagation in bulk flow



RAYLEIGH STREAMING - Caused by sound absorption near a boundary, either when sound reaches a boundary, or when a boundary is vibrating in a still medium.

Governing equations:

$$\rho_0 \left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{U} \right] = -\nabla P + \eta \Delta \mathbf{U} + \mathbf{f} \,,$$

$$\nabla \cdot \mathbf{U} = 0 \,,$$

$\mathbf{U} = \langle \rho \mathbf{v} \rangle / \rho_0$ – streaming velocity,

$\mathbf{f} = -\rho_0 \left\langle (\mathbf{v}_{ac} \cdot \nabla)\mathbf{v}_{ac} + \mathbf{v}_{ac} (\nabla \cdot \mathbf{v}_{ac}) \right\rangle$

– radiation force per unit mass of liquid,

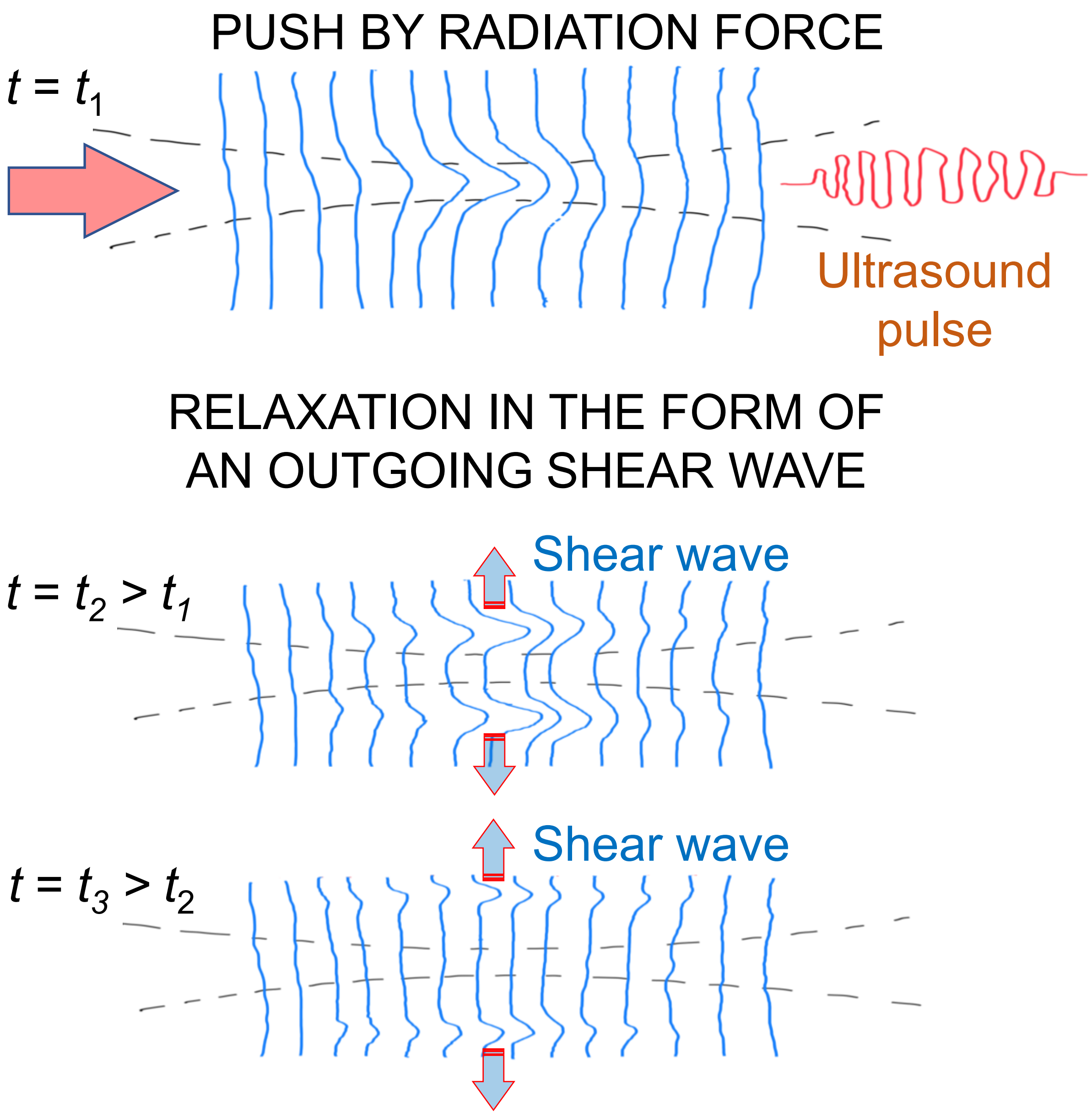
$\langle \ldots \rangle$ – averaging over the wave period.

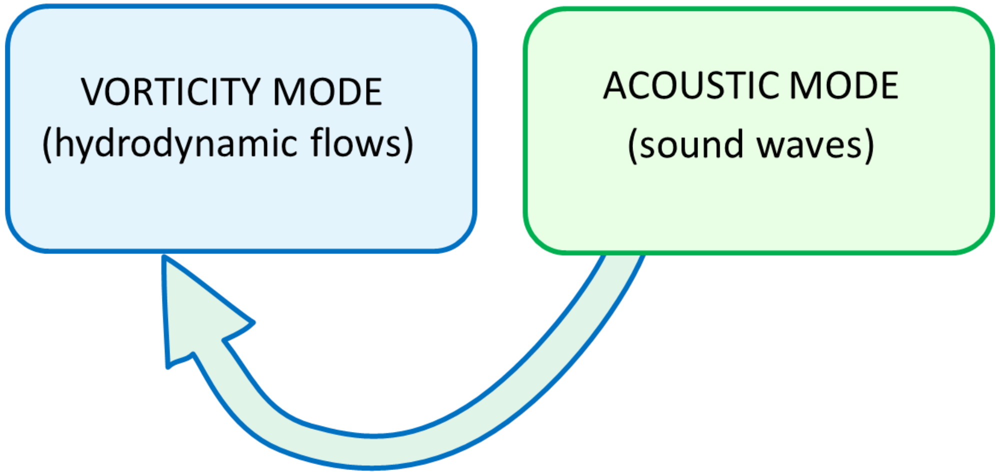
For a harmonic wave of arbitrary spatial structure:

$$\mathbf{f} = \frac{2\alpha}{c_0} \mathbf{I}$$

$\mathbf{I} = \langle p_{ac} \mathbf{v}_{ac} \rangle$ – wave intensity

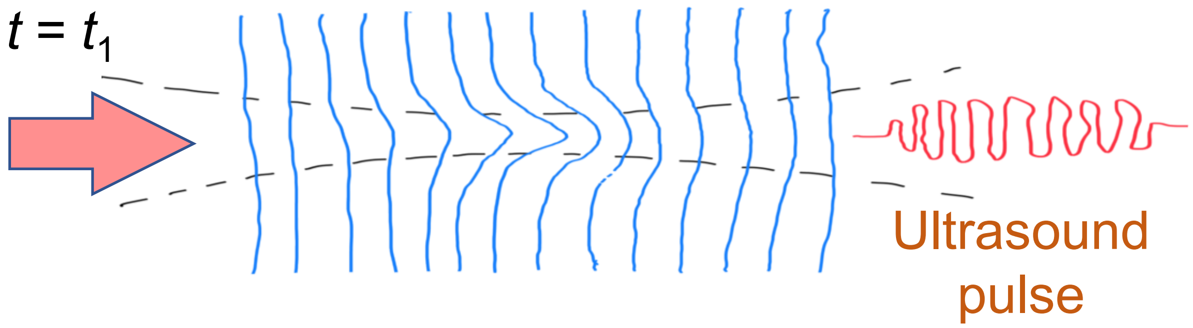
SHEAR WAVE GENERATION IN SOFT SOLIDS



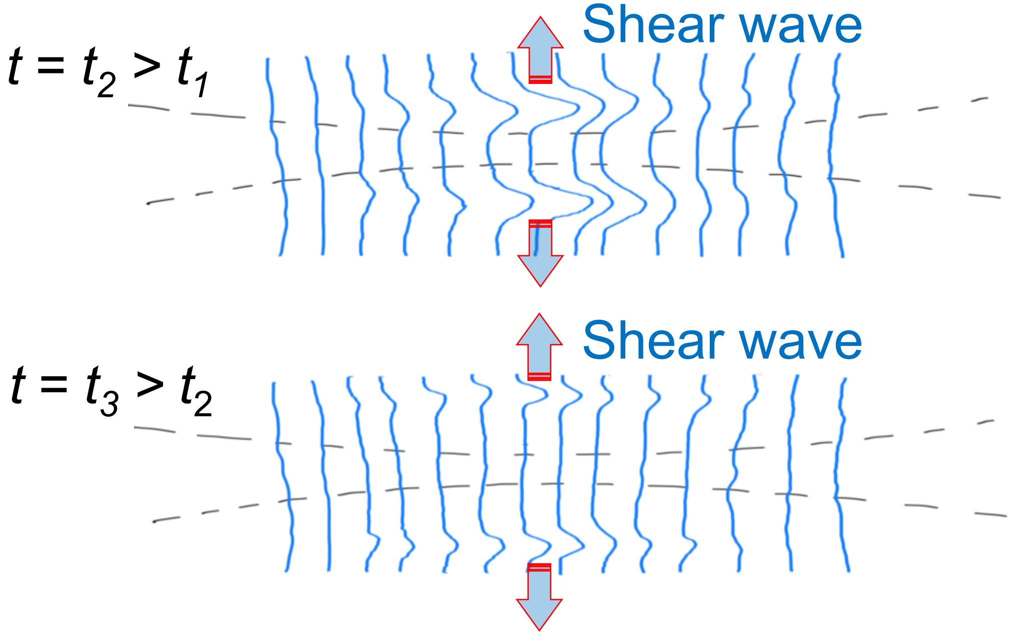


SHEAR WAVE GENERATION IN SOFT SOLIDS

PUSH BY RADIATION FORCE

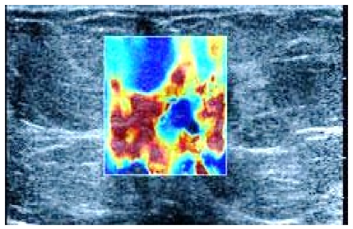


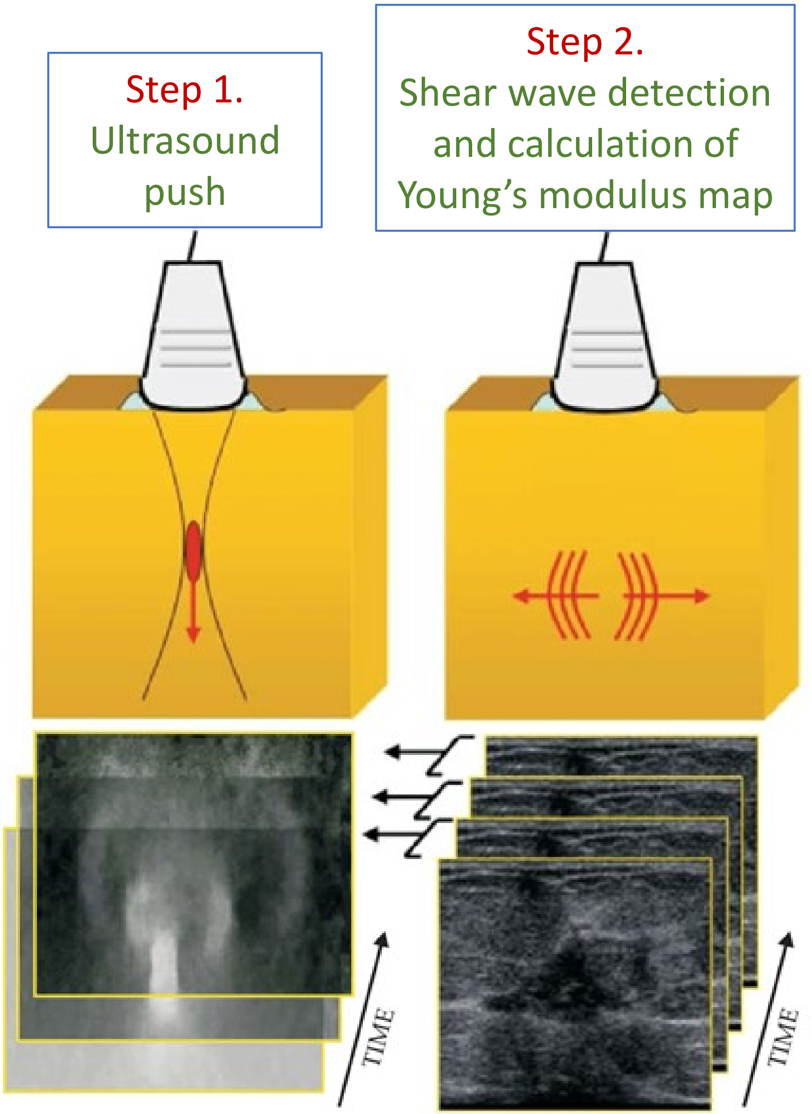
RELAXATION IN THE FORM OF AN OUTGOING SHEAR WAVE



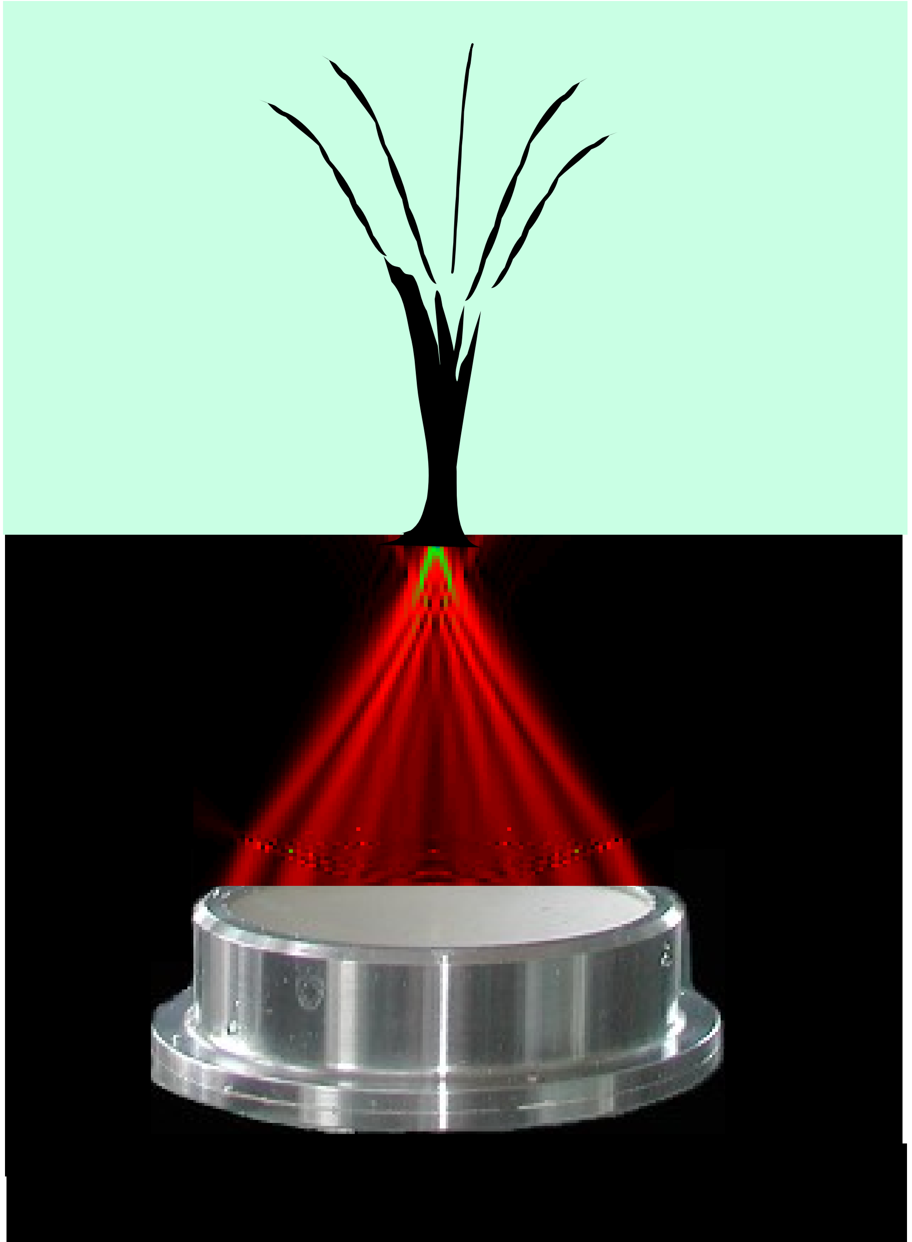
Shear wave elastography

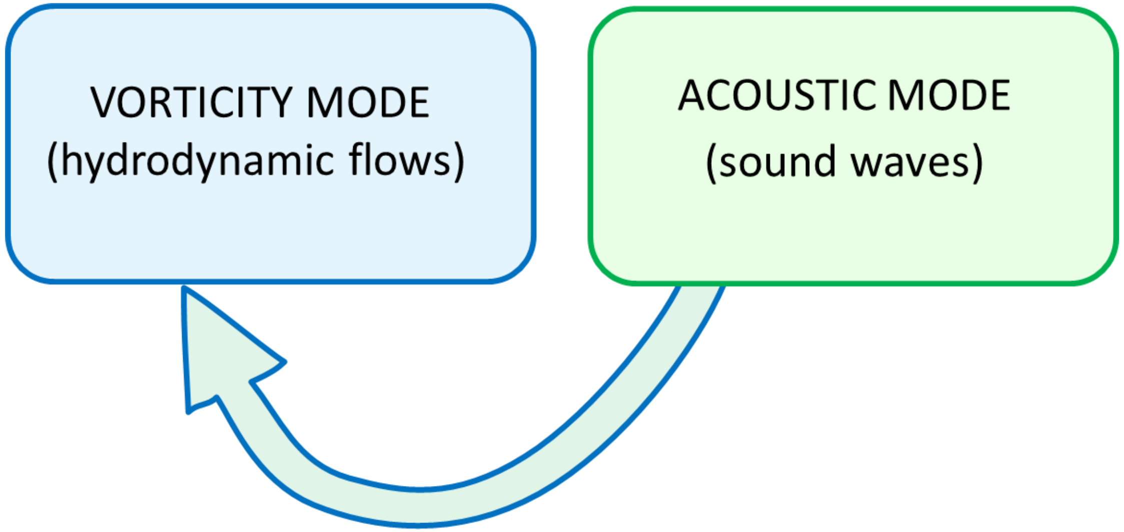
«Aixplorer» Supersonic Imagine



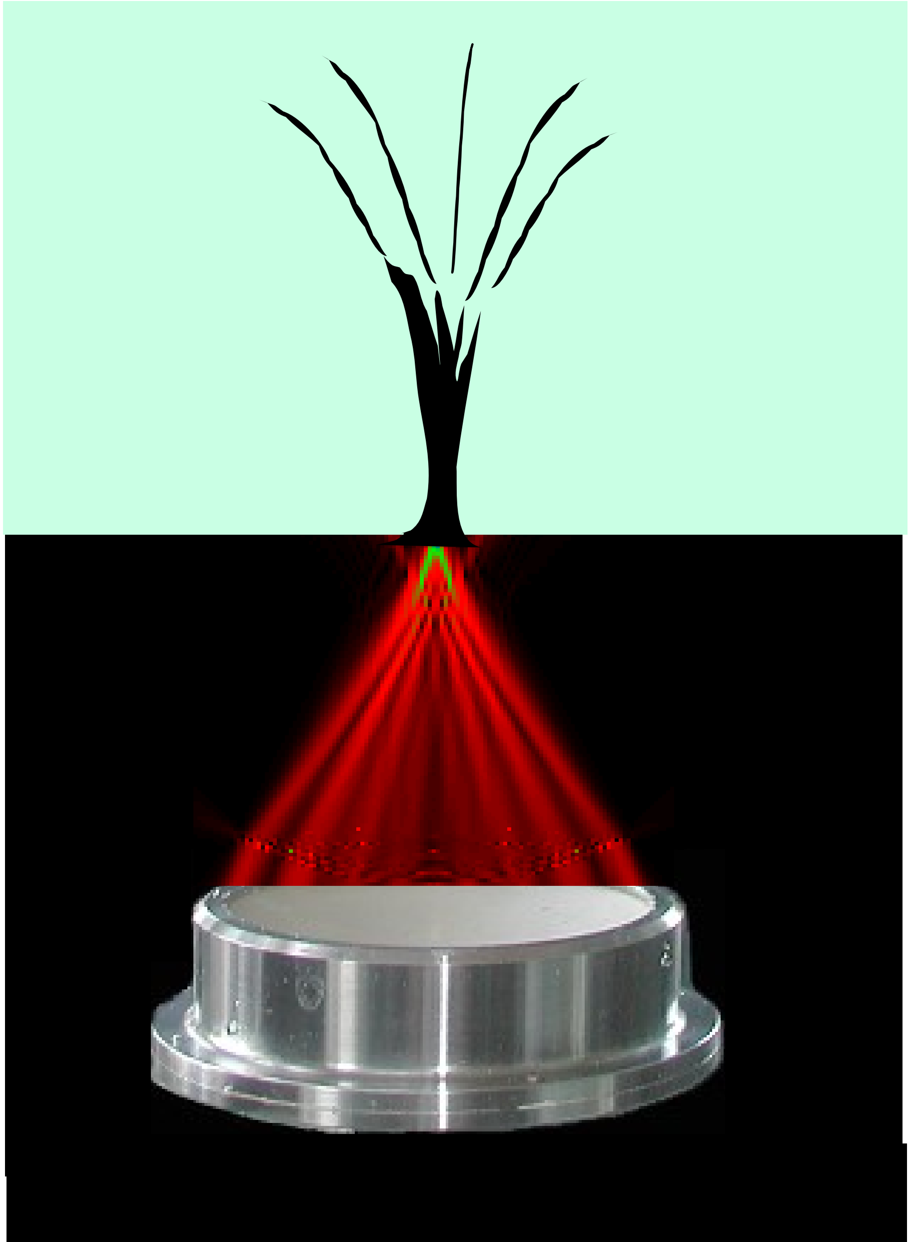


ACOUSTIC FOUNTAIN





ACOUSTIC FOUNTAIN



HIFU-induced acoustic fountain

HIFU-induced acoustic fountain and atomization

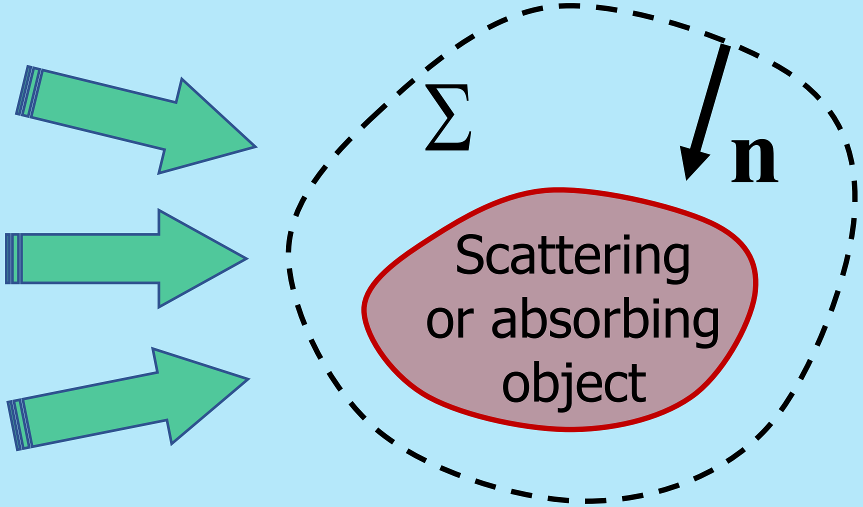
RADIATION FORCE ACTING ON ABSORBERS AND SCATTERERS

Momentum flux tensor (after Langevin):

$$T_{ij} = \left(\frac{p^2}{2\rho_0 c_0^2} - \frac{\rho_0 \mathbf{v}\cdot\mathbf{v}}{2} \right)\delta_{ij} + \rho_0 v_i v_j \,,$$

$$\mathbf{v} = (v_1, v_2, v_3)$$

Radiation force: $F_i = -\oiint\limits_{\Sigma} \langle T_{ij} \rangle n_j ds$



Acoustic pressure in CW case: $p' = \mathrm{Re}\left(P \cdot e^{-i\omega t}\right)$

Particle velocity complex amplitude: $\mathbf{V} = \nabla P / (i\rho\omega)$

Radiation force as a functional of a CW acoustic field:

$$\mathbf{F} = \oiint\limits_{S} \left\{ \left(\frac{\rho |\mathbf{V}|^2}{4} - \frac{|P|^2}{4\rho c^2} \right)\mathbf{n} - \frac{\rho}{2}\mathrm{Re}\left[\mathbf{V}^* (\mathbf{V}\cdot\mathbf{n})\right] \right\} dS$$

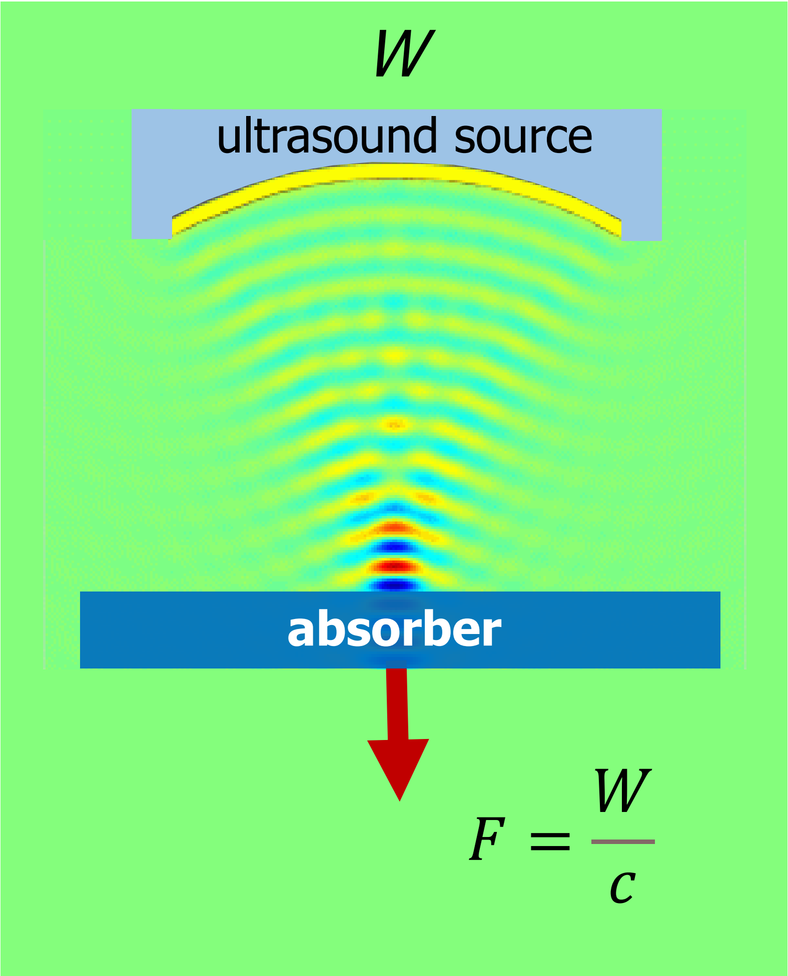
RADIATION FORCE ACTING ON AN EXTENDED ABSORBER

$$\mathbf{F} = \oiint_S \left\{ \left(\frac{\rho|\mathbf{V}|^2}{4} - \frac{|P|^2}{4\rho c^2} \right)\mathbf{n} - \frac{\rho}{2}\mathrm{Re}\left[\mathbf{V}^*(\mathbf{V}\cdot\mathbf{n})\right] \right\} dS$$

Plane wave case: $\mathbf{V} = -\dfrac{P}{\rho c}\mathbf{n}$

$$F = \iint_{\substack{\text{absober}\\ \text{surface}}} \frac{|P|^2}{2\rho c^2} dS \Rightarrow \boxed{F = \frac{W}{c}}$$

W - total acoustic power



Change of weight / Acoustic power

68 mg / watt , i.e., 15 watt $\leftrightarrow$ 1 g

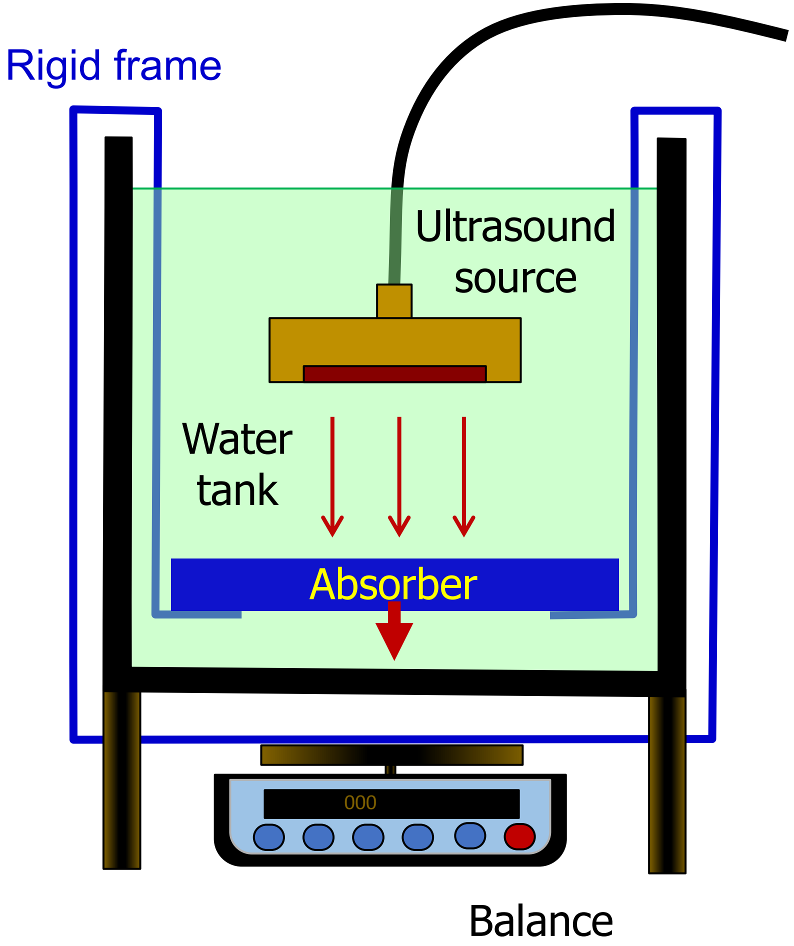
Interpretation in terms of phonon energy and momentum

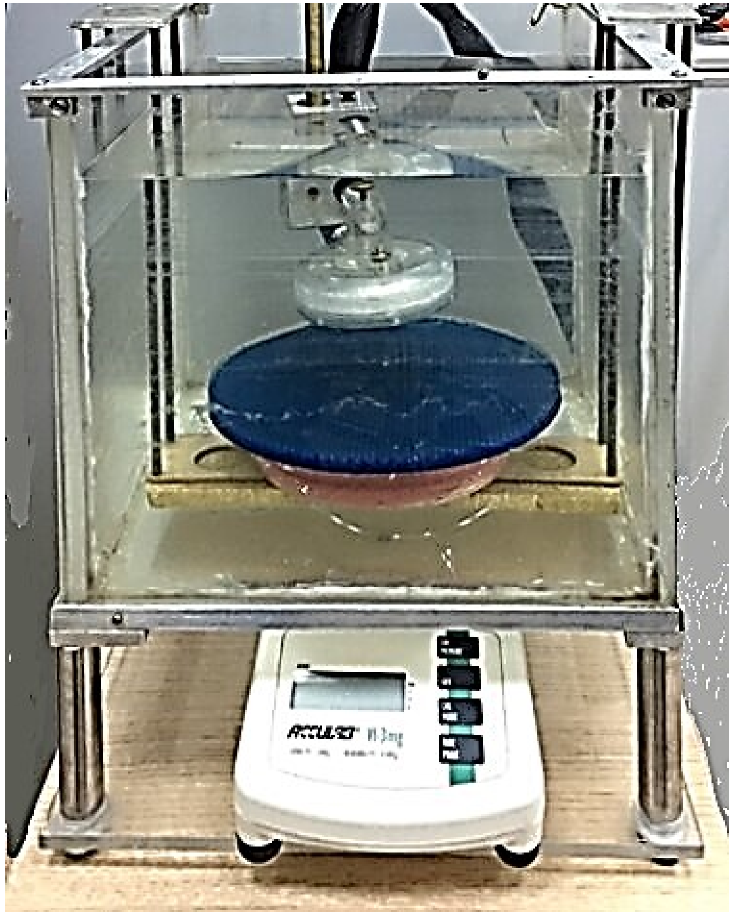
Quasi-momentum: $\boxed{\Delta N = \hbar k} = \hbar\dfrac{\omega}{c} = \dfrac{\Delta E}{c}$

Energy: $\boxed{\Delta E = \hbar\omega}$

$$\Delta N/\Delta t = \frac{\Delta E/\Delta t}{c} \Rightarrow \boxed{F = \frac{W}{c}}$$

RADIATION FORCE BALANCE



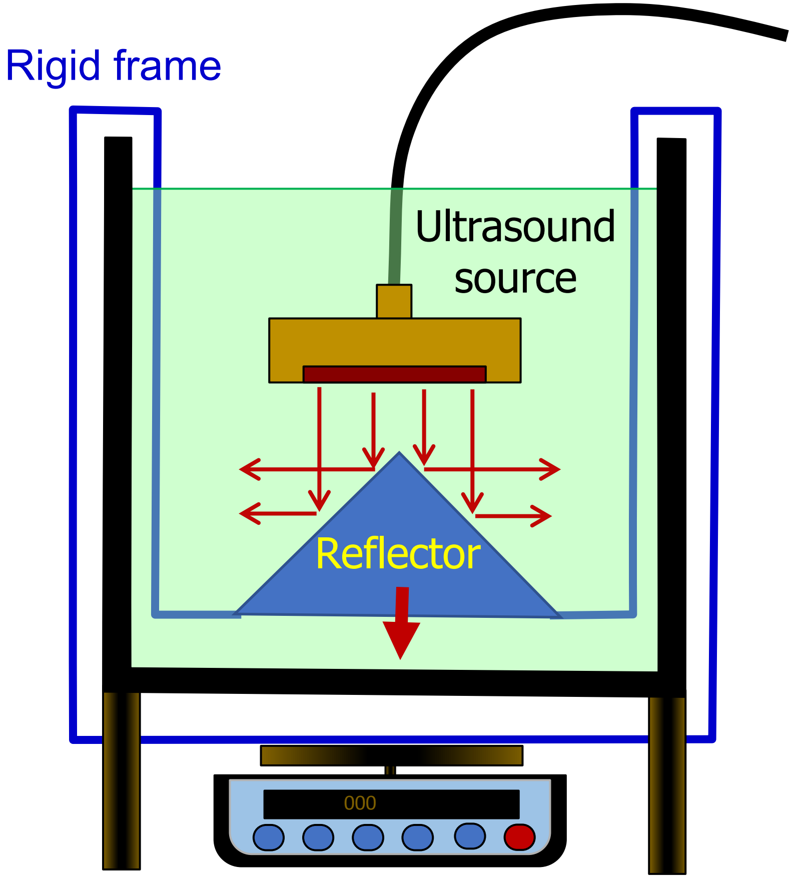


Measuring transducer power 2 weeks ago, in Seattle …

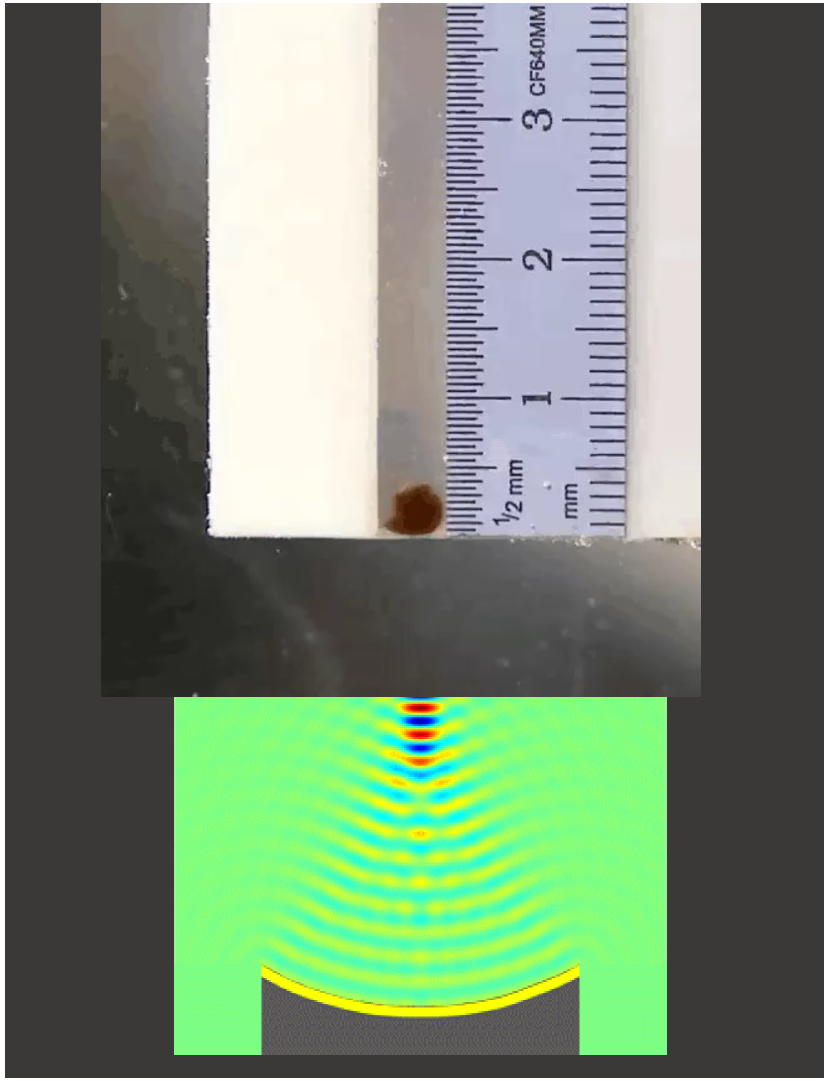
IEC STANDARD: IEC61161:2006, Ed.2. Ultrasonics – Power measurement – Radiation force balances and performance requirements, International Electrotechnical Commission, Geneva, 2006.

RADIATION FORCE ACTING ON A REFLECTOR (SCATTERER)

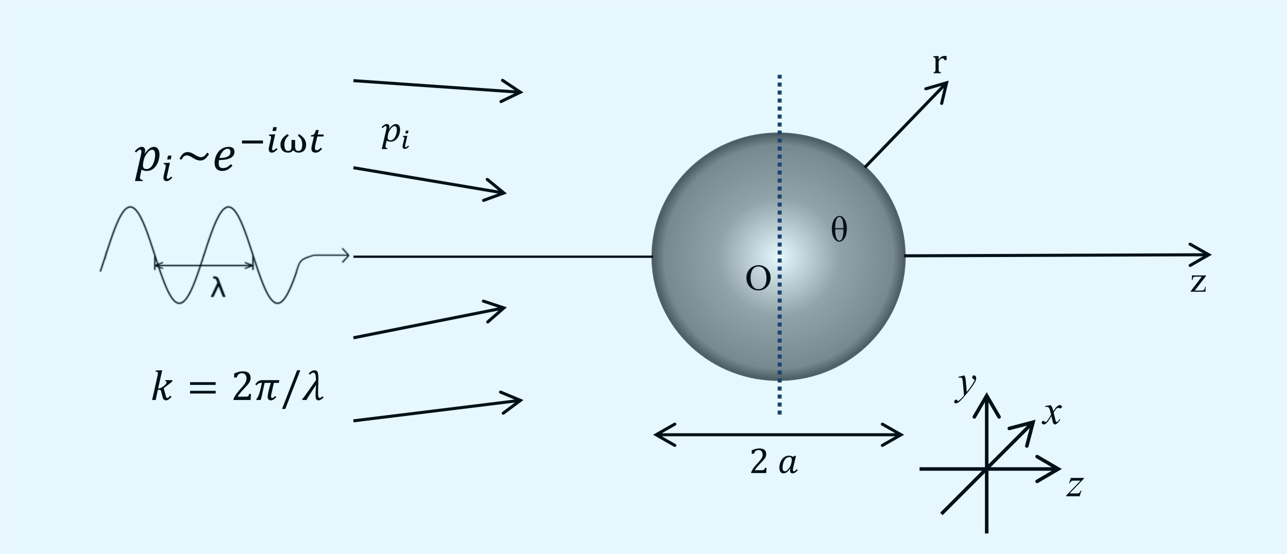
Radiation force balance
with conical reflector

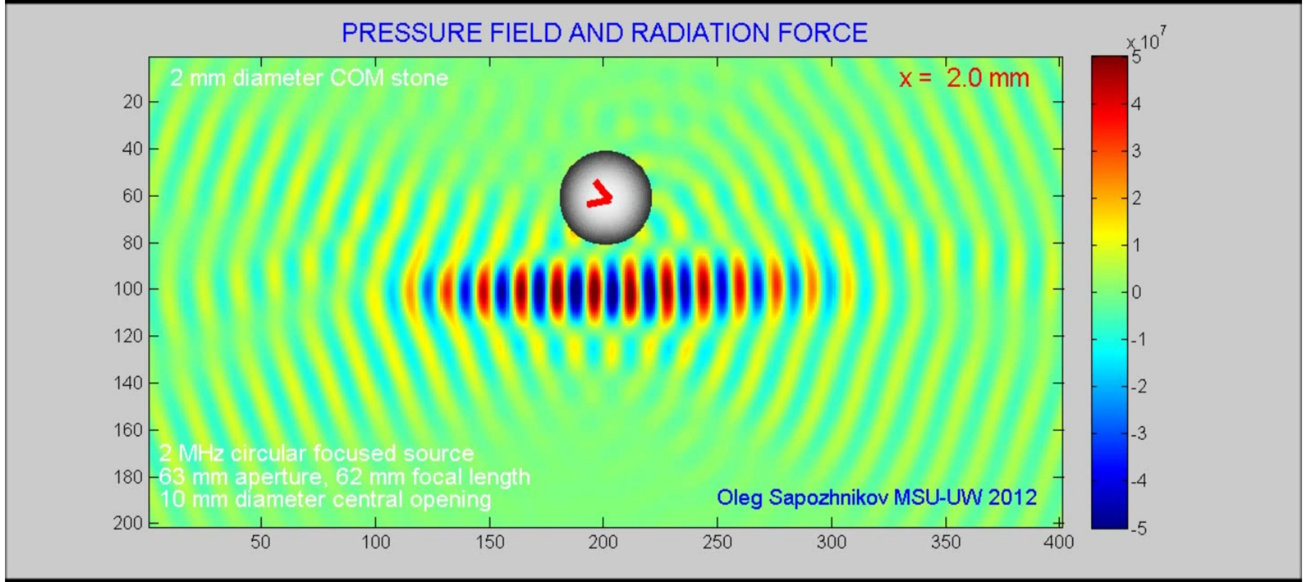


Repulsion of kidney stones



Radiation force of an arbitrary acoustic beam on an elastic sphere in a fluid





$$S(k_x,k_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dxdy\; p_{inc}(x,y,0)e^{-ik_x x - ik_y y}$$

$$H_{nm} = \iint_{k_x^2+k_y^2<k^2} dk_x dk_y S(k_x,k_y) Y_{nm}^*(\theta_k,\varphi_k)$$

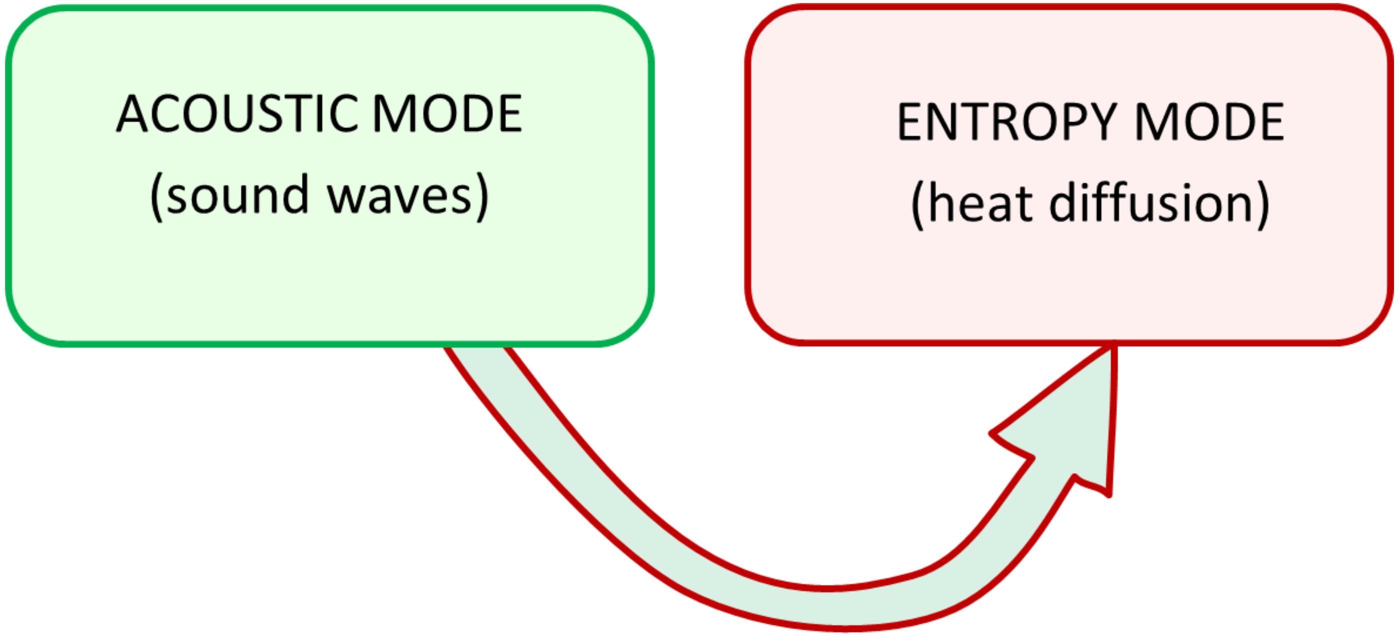
$$F_z = -\frac{1}{4\pi^2\rho c^2 k^2}\mathrm{Re}\left\{\sum_{n=0}^{\infty}\Psi_n \sum_{m=-n}^{n} B_{nm} H_{nm} H_{n+1,m}^*\right\}$$

$$\Psi_n = (1+2c_n)(1+2c_{n+1}^*)-1$$

$$B_{nm} = \sqrt{\frac{(n+m+1)(n-m+1)}{(2n+1)(2n+3)}}$$

Sapozhnikov, Bailey, JASA 2013

Excitation of the entropy mode by sound (transfer of energy from sound to medium – heating by acoustic waves)



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$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0$$

$$\boxed{\rho T\left[\frac{\partial s}{\partial t}+(\mathbf{v}\cdot\nabla)s\right]=\kappa\,\Delta T+\frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right)^2+\varsigma(\nabla\cdot\mathbf{v})^2}$$

Thermodynamic relations

$$d\rho=dp/c^2-ds\cdot\alpha_V\,T\rho/c_p$$

$$dT=dp\cdot\alpha_V\,T/(\rho c_p)+ds\cdot T/c_p$$

Fluid parameters

$$\alpha_V=-\rho^{-1}(\partial\rho/\partial T)_p$$

$$c=\sqrt{(\partial p/\partial\rho)_s}\qquad c_p=T(\partial s/\partial T)_p$$

BASIC EQUATIONS FOR THE HEAT PRODUCTION

Second law of thermodynamics $\delta Q = TdS$ corresponds to the following entropy balance equation:

$$\frac{\partial(\rho s)}{\partial t} + \nabla \cdot \mathbf{S} = \frac{1}{T}\frac{\partial Q}{\partial t}$$

$\mathbf{S} = \rho s \mathbf{v} + \dfrac{\mathbf{q}}{T}$ is entropy flux, where $\mathbf{q} = -\kappa \nabla T$ is Fourier's law. Equations of fluid dynamics provide the following amount of heat produced in the fluid per unit volume and time:

$$\frac{\partial Q}{\partial t} = \frac{\kappa}{T}(\nabla T)^2 + \frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla \cdot \mathbf{v}\right)^2 + \varsigma(\nabla \cdot \mathbf{v})^2$$

For a plane wave the energy dissipation (averaged over the wave period) per unit volume and time is

$$\left\langle \frac{\partial Q}{\partial t} \right\rangle_{\substack{\text{plane}\\ \text{wave}}} = \frac{\delta}{\rho_0 c_0^4}\left\langle \left(\frac{\partial p'}{\partial t}\right)^2 \right\rangle$$

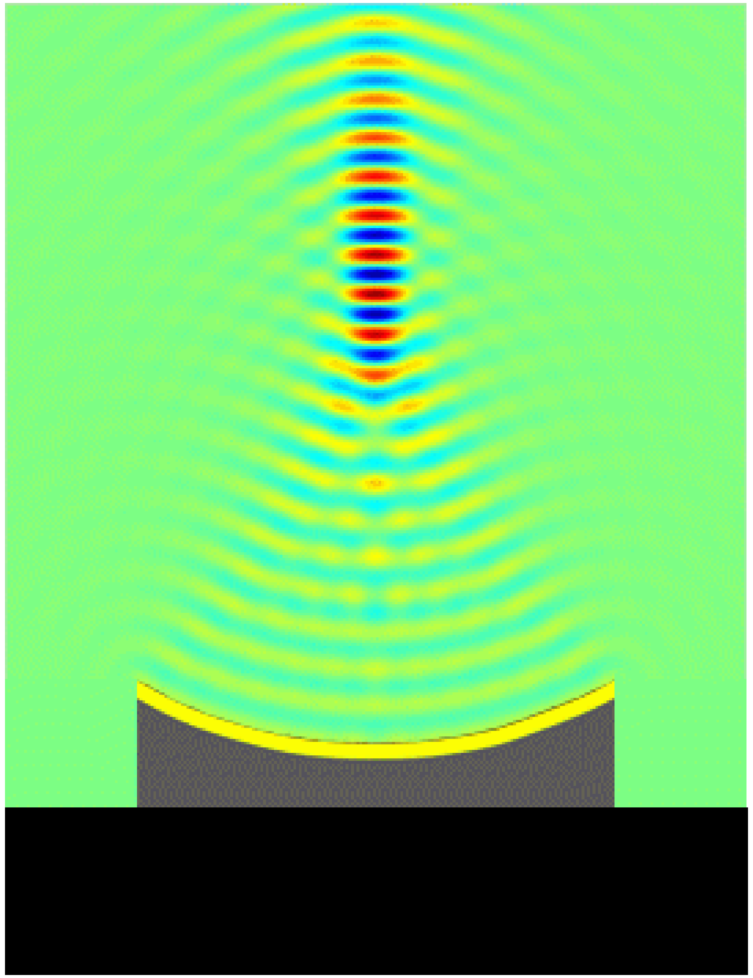
Here $\delta = \left[4\eta/3 + \varsigma + \kappa\left(c_v^{-1} - c_p^{-1}\right)\right]/\rho_0$ is the diffusivity coefficient. For a particular case of a propagating harmonic wave $I = \langle p'^2 \rangle / (\rho_0 c_0)$ is wave intensity, which gives

$$\left\langle \frac{\partial Q}{\partial t} \right\rangle_{\substack{\text{plane}\\ \text{sine wave}}} = 2\alpha I$$

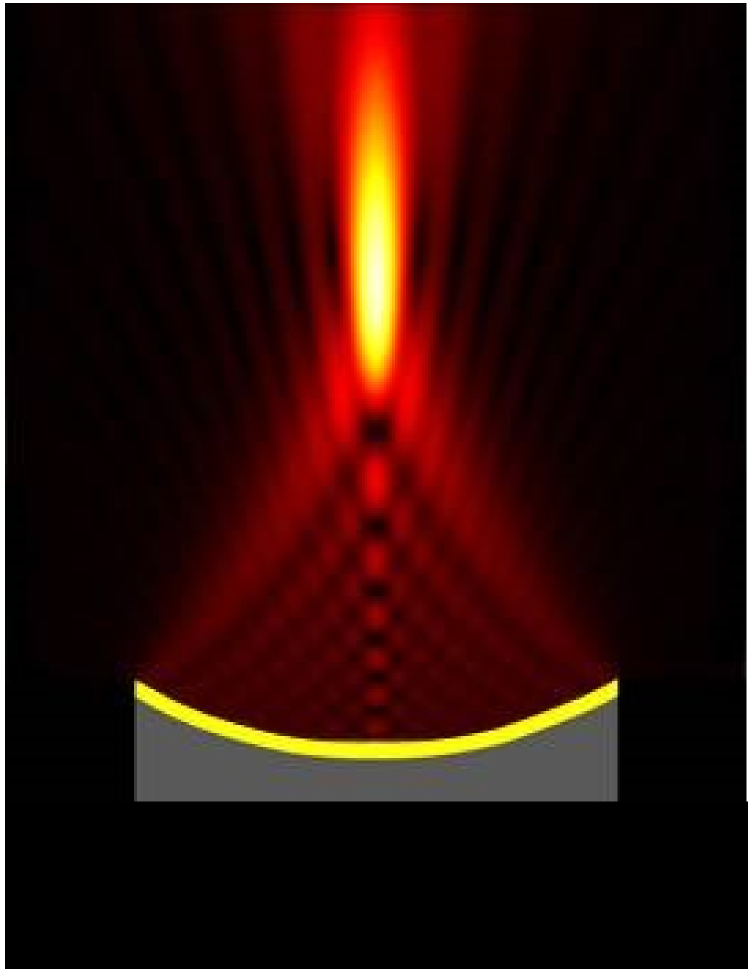
Acoustic field of a focused ultrasound source and associated heating

$$\left\langle \frac{\partial Q}{\partial t} \right\rangle_{\substack{\text{plane} \\ \text{sine wave}}} = 2\alpha I$$

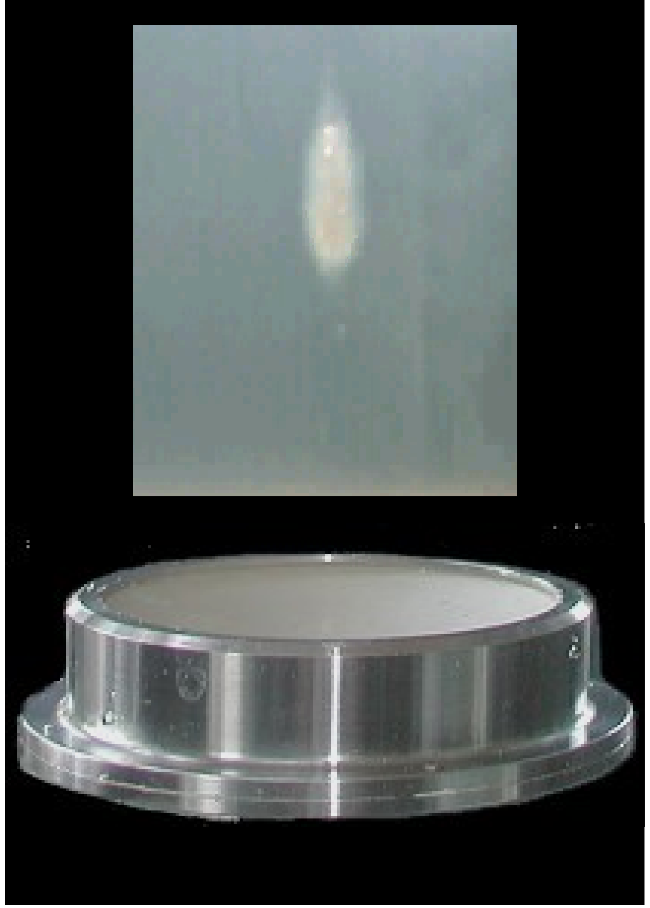
Acoustic pressure, p'



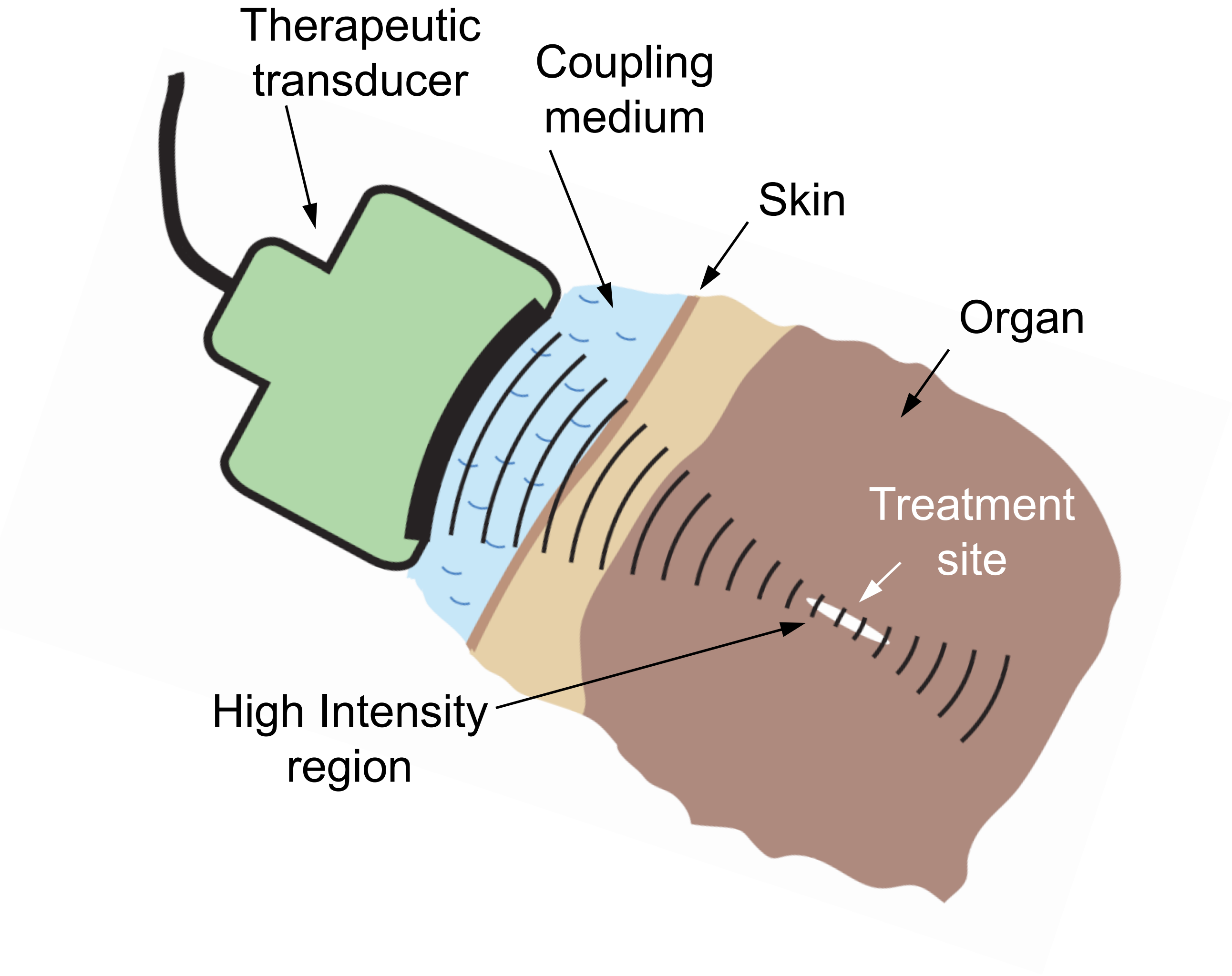
Intensity, $I = \langle p'^2 \rangle / (\rho_0 c_0)$

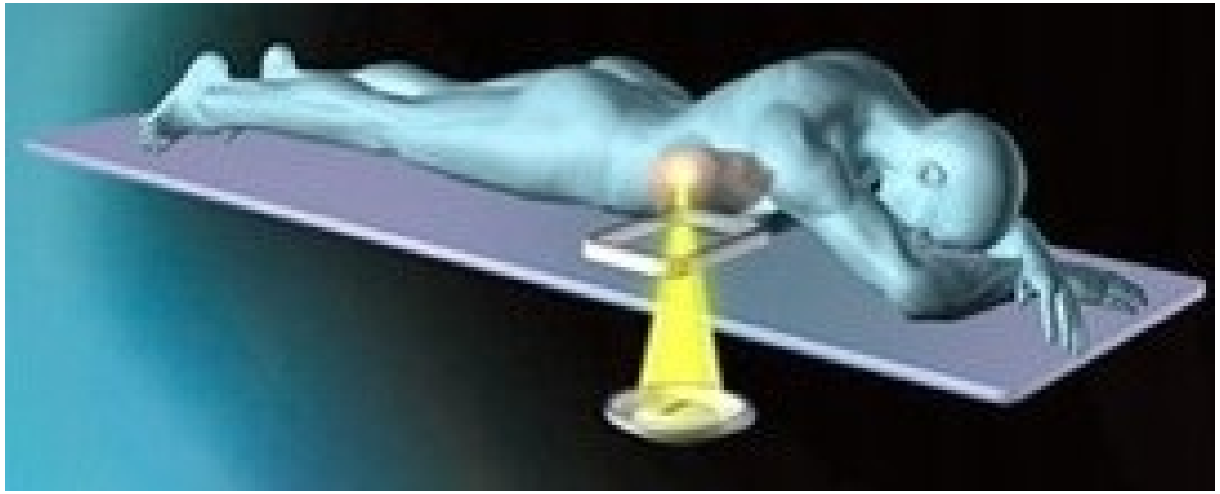


Gel damage due to heating

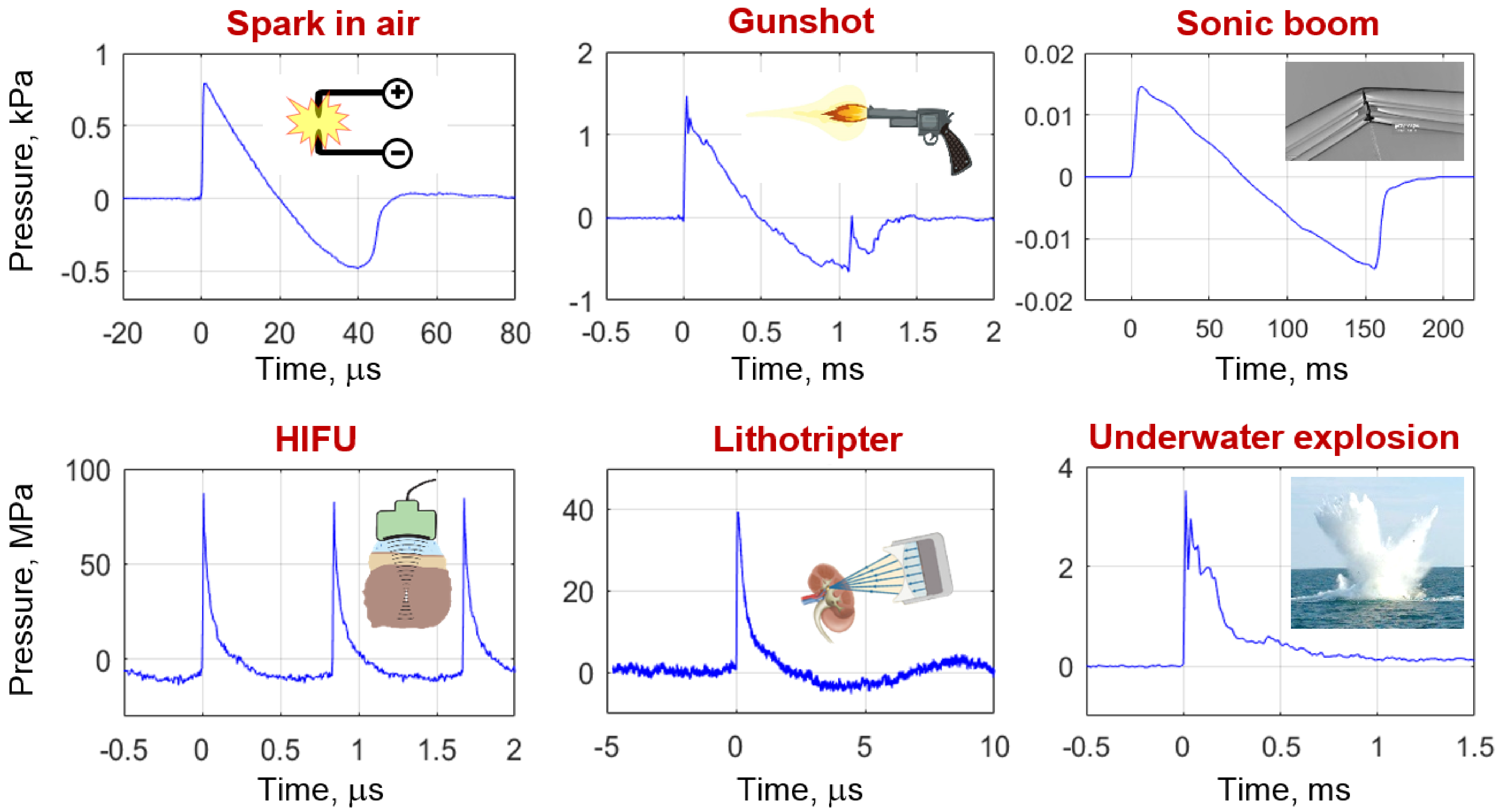


Use of high-intensity focused ultrasound (HIFU) for therapy and surgery





Weak shock waves resulting from nonlinear propagation



Entropy (and therefore, heat) production at the shock front

$$\Delta s = \frac{\beta}{6\rho_0^3 c_0^4 T_0} (\Delta p)^3$$

$$Q_{\text{shock}} = \rho_0 T_0 \Delta s = \frac{\beta}{6\rho_0^2 c_0^4} (\Delta p)^3$$

$$\left\langle \frac{\partial Q}{\partial t} \right\rangle_{\substack{\text{sawtooth} \\ \text{wave}}} = \frac{\beta f_0}{6\rho_0^2 c_0^4} (\Delta p)^3$$

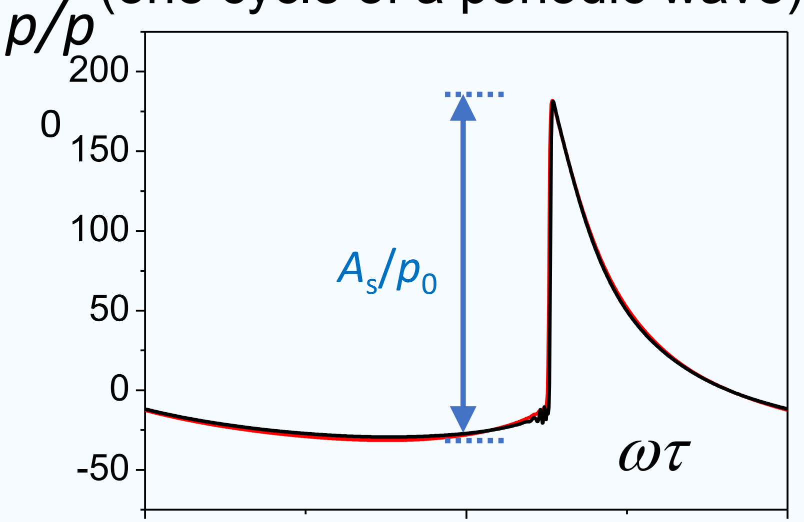
$$\left\langle \frac{\partial Q}{\partial t} \right\rangle_{\substack{\text{plane} \\ \text{sine wave}}} = 2\alpha I = \alpha \frac{A_0^2}{\rho_0 c_0}$$

$$\frac{\langle \partial Q / \partial t \rangle_{\substack{\text{sawtooth} \\ \text{wave}}}}{\langle \partial Q / \partial t \rangle_{\text{sine wave}}} = \frac{2}{3} \frac{\beta f_0}{\alpha \rho_0 c_0^3} \Delta p \;\; >> 1 \;\; \text{for large} \;\; \Delta p$$

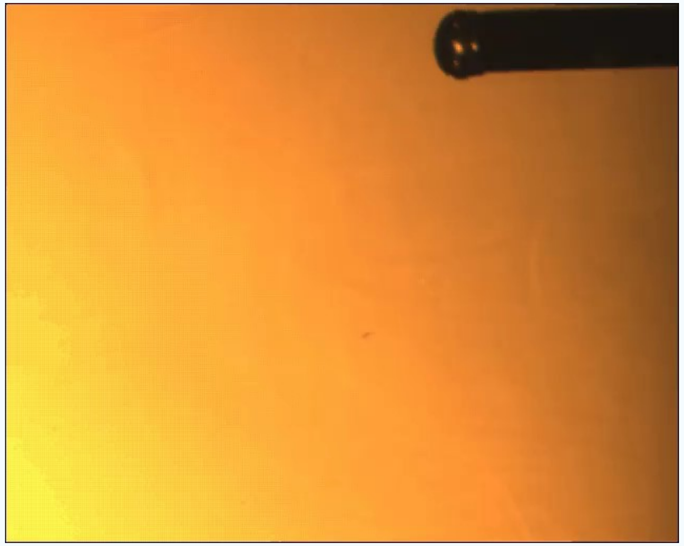
Fast boiling at the HIFU focus in shockwave regime

Canney *et. al*. UMB, v.36 (2010)

Focal waveforms in water (one cycle of a periodic wave)

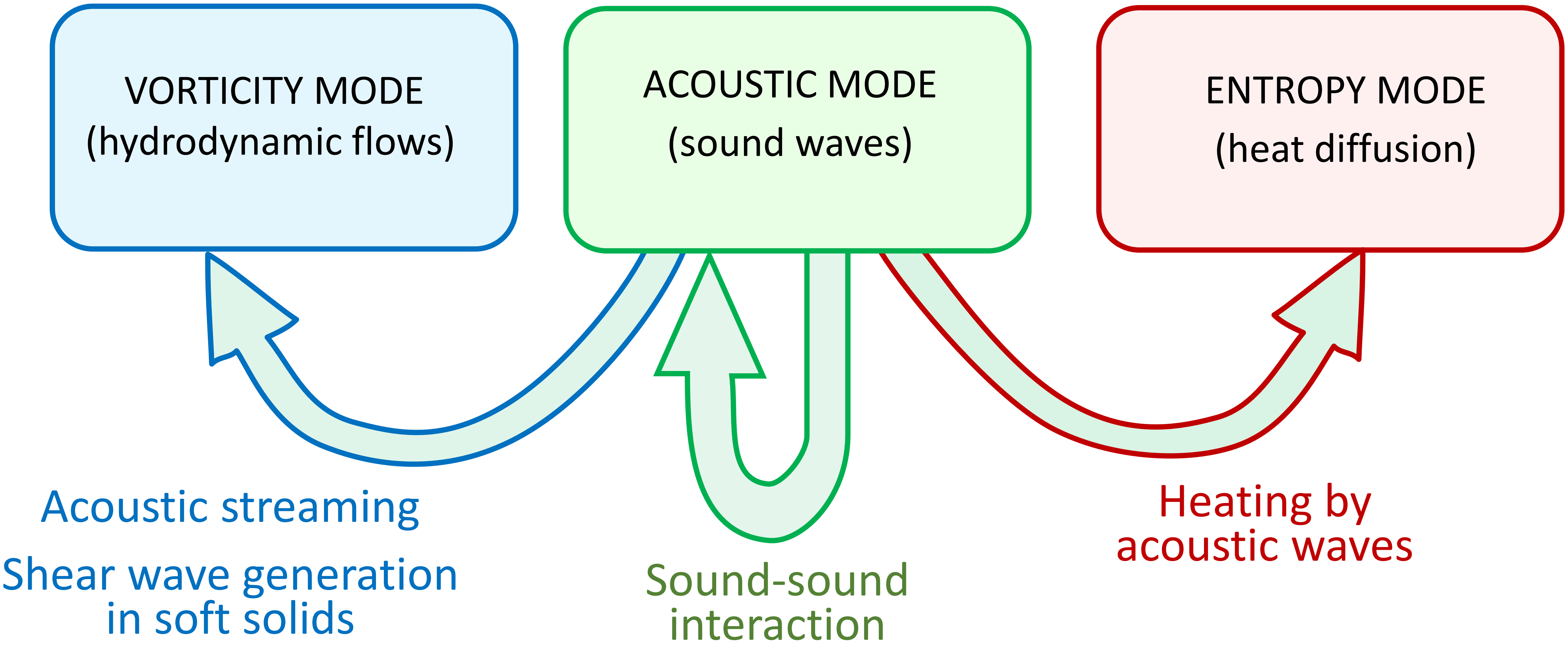


Gel near the focus



Tissue at the focus can reach 100°C in a millisecond!

THE SUBJECTS OF STUDY IN NONLINEAR ACOUSTICS



SHORT COURSE ON NONLINEAR ACOUSTICS - to be continued

David Blackstock (1930–2021) in Seattle, 2020

WHAT TO READ

1. Hamilton, M.F., and Blackstock, D.T. (Eds.). *Nonlinear Acoustics*. – Acoustical Society of America, Melville, NY, 2008.
2. Rudenko, O.V., and Soluyan, S.I. *Theoretical Foundations of Nonlinear Acoustics*. – Consultants Bureau, London, 1977.
3. Naugolnykh. K., and Ostrovsky, L. *Nonlinear Wave Processes in Acoustics*. – Cambridge Univ. Press, Cambridge, 1998.
4. Pierce, A.D. *Acoustics: An Introduction to Its Physical Principles and Applications*. – Acoust. Soc. Am., 1994.
5. Landau, L.D., and Lifshits, E.M. *Fluid Mechanics*. – 2nd ed., Pergamon, New York, 1987.
6. Chu,B.-T., and Kovasznay, L.S.G. Non-linear interactions in a viscous heat-conducting compressible gas. – Journal of Fluid Mechanics **3**, 494–514 (1958).
7. Pierce, A.D. Entropy, acoustic, and vorticity mode decomposition as an initial step in nonlinear acoustics formulations. - Nonlinear Acoustics at the Beginning of the 21st Century, edited by O.V. Rudenko and O.A. Sapozhnikov (Faculty of Physics, MSU, Moscow, 2002), vol. 1, pp. 11–19.
8. Nyborg, W.L. Acoustic streaming, in *Physical Acoustics* (W. P. Mason, ed.), vol. 2, Pt B, ch. 11, 265–331, New York, NY: Academic Press, 1965.
9. Lighthill, J. Acoustic streaming. J. Sound Vib. **61**, 391–418 (1978).
10. McIntyre, M.E. On the ‘wave momentum’ myth. – J. Fluid Mech. **106**, 331-347 (1981).
11. Sapozhnikov, O.A., and Bailey, M.R. Radiation force of an arbitrary acoustic beam on an elastic sphere in a fluid. – J. Acoust. Soc. Am. **133**(2), 661-676 (2013).
12. Sapozhnikov, O.A. High-intensity ultrasonic waves in fluids: Nonlinear propagation and effects. - In: Power Ultrasonics. Applications of High-intensity Ultrasound, ed. by Gallego-Juarez, J.A., and Graff, K.F., Chapter II, Woodhead Publishing Series in Electronic and Optical Materials (Elsevier), Cambridge, №66, pp. 9-35 (2014).